

## LIGHT CONES NEAR THE EVENT HORIZON

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 14; Problem 14.4.

For an object moving in the Schwarzschild metric, its proper time interval is given by

$$d\tau^2 = -ds^2 = \left(1 - \frac{2GM}{r}\right) dt^2 - \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 \quad (1)$$

If the object is moving radially, then  $d\theta = d\phi = 0$  and since  $d\tau$  must be a real number, we must have  $d\tau^2 \geq 0$  so

$$\left(1 - \frac{2GM}{r}\right) dt^2 - \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 \geq 0 \quad (2)$$

By taking the equality in this formula, we can find the equation of the light cone for various values of  $r$ . In general, we have

$$\left(\frac{dt}{dr}\right)^2 = \left(1 - \frac{2GM}{r}\right)^{-2} \quad (3)$$

For various values of  $r$ , we get

$r$	$\left(\frac{dt}{dr}\right)^2$	$\frac{dt}{dr}$
$4GM$	4	$\pm 2$
$3GM$	9	$\pm 3$
$\frac{5}{2}GM$	25	$\pm 5$
$\frac{3}{2}GM$	9	$\pm 3$
$GM$	1	$\pm 1$
$\frac{1}{2}GM$	$\frac{1}{9}$	$\pm \frac{1}{3}$

To convert these results into plots of the light cone, we must remember that if  $r > 2GM$ , increasing proper time corresponds to increasing  $t$ , while for  $r < 2GM$ , space and time swap round, so increasing proper time corresponds to *decreasing*  $r$ . If we plot the light cone on a 2-d graph of  $t$  versus  $r$ , then for  $r > 2GM$ , the light cone opens upwards and gets progressively narrower as  $r$  approaches  $2GM$  from above.

For  $r < 2GM$ , the light cone opens to the left, starting with a very wide (effectively  $180^\circ$ ) angle between the two sides, and then getting narrower as  $r \rightarrow 0$ , with the cone becoming a spike that points to the left along the  $r$  axis.