

ESCAPE VELOCITY NEAR AN EVENT HORIZON

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 14; Problem 14.5.

In Newtonian physics, the escape velocity of an object of mass m starting from a distance R from another mass M is defined as the velocity which the object must have if it is to arrive at infinity with a velocity of zero. In other words, the total energy E of the object (kinetic plus potential) must be zero at all times (since energy is conserved). The escape velocity v_e therefore satisfies

$$(1) \quad \frac{1}{2}mv_e^2 = \frac{GMm}{R}$$

$$(2) \quad v_e = \sqrt{\frac{2GM}{R}}$$

It turns out that the escape velocity calculated in the Schwarzschild metric obeys the same formula. An object at rest at infinity has a total energy per unit mass $e = 1$. We've also seen that the velocity of an object that moves radially as observed by an observer at R is

$$(3) \quad v_{obs}^2 = 1 - \frac{1}{e^2} \left(1 - \frac{2GM}{R} \right)$$

With $e = 1$, this gives the same formula for escape velocity

$$(4) \quad v_{obs} = v_e = \sqrt{\frac{2GM}{R}}$$

This formula applies only for $R > 2GM$, that is, outside the event horizon, and it predicts that as $R \rightarrow 2GM$, $v_e \rightarrow 1$, which is the basis for saying that a black hole will suck in anything that crosses the event horizon, so nothing with a non-zero rest mass can have a speed as great as 1.