KRUSKAL-SZEKELES METRIC: WHAT CAN YOU SEE AS YOU FALL INTO A BLACK HOLE?

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 15; Problem 15.4.

The Kruskal-Szekeres (KS) metric is an alternative to the Schwarzschild (S) metric that makes many phenomena easier to visualize by drawing a diagram in which world lines are plotted on a graph of the coordinate v versus u. For radial motion, the KS metric is

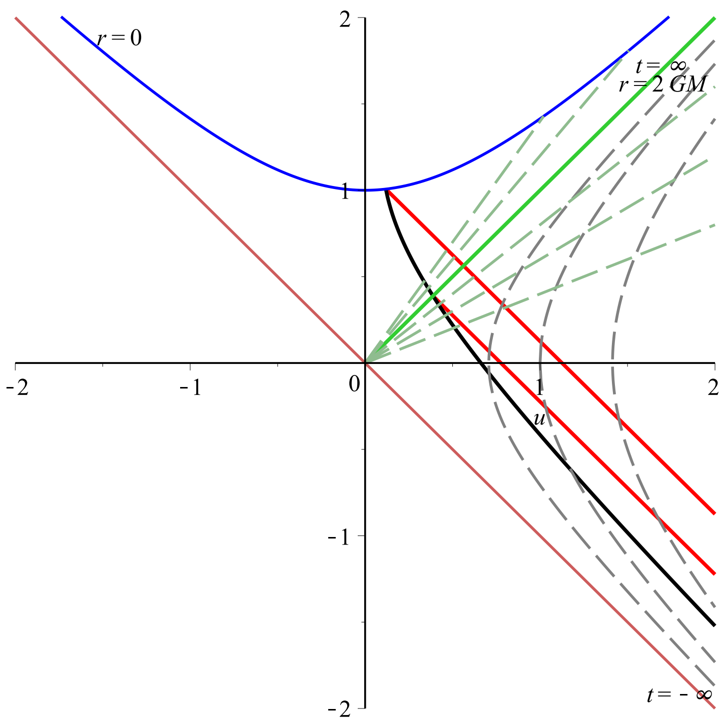
$$ds^2=\frac{32\left(GM\right)^3}{r}e^{-r/2GM}\left(du^2-dv^2\right) \tag{0.1}$$

and the relation between the KS coordinates u and v and the S coordinates r and t is

$$u^2-v^2 = \left(\frac{r}{2GM}-1\right)e^{r/2GM} \tag{0.2}$$

$$2GM\ln\left|\frac{u+v}{u-v}\right| = t \tag{0.3}$$

As an example of using a KS diagram, suppose we have an observer in a space ship that is falling into a black hole. As we saw in the last post, any massive object's world line must have a slope with a magnitude greater than 1 at all of its points in the KS diagram. One possible world line is shown as the heavy black curve in the diagram:



The motion of the observer is upwards in the diagram. In S coordinates, an observer at infinity sees the ship slow down as it approaches the event horizon, eventually freezing at the horizon, since the S t coordinate becomes infinite there (as shown in the diagram by the heavy green line in the upper right quadrant). However, as measured by the observer in the ship, it takes a finite timeto move from a point outside the horizon to a point inside it; there is no divergence at $r = 2GM$. This is shown in the KS diagram as the black curve simply continues across the green line without anything unusual happening.

Does the fact that the ship crosses the $t = \infty$ line mean that the observer in the ship can see the entire future of the outside world as he crosses $r = 2GM$? To answer this, we first note that in order for the observer to see an event from the outside world, a photon from that event must be able to reach him before he gets to $r = 0$, where his own world line ends. Since photons travel along lines with slopes of ± 1, the only photons that can reach him after he crosses the horizon are those between the two heavy red lines shown. Thus he can see into the distant future (that is, very large t values) only for events that happen just outside the horizon (that is, events just to the lower right of the green line, between the two red lines). Events that occur at $t = 0$ lie on the u axis, so he can see only those events that lie between approximately $u = 0.75$ and $u = 1.1$. The r values of these events are determined by the set of hyperbolas that cross the u axis between these points (some of the hyperbolas of constant r are shown as dashed grey curves).

Thus even though he is inside the event horizon, the observer can still see some events that happened outside the horizon, but he cannot see all events that happen at any time in the future.