KRUSKAL-SZEKERES DIAGRAMS: SAVING A SPACE SHUTTLE

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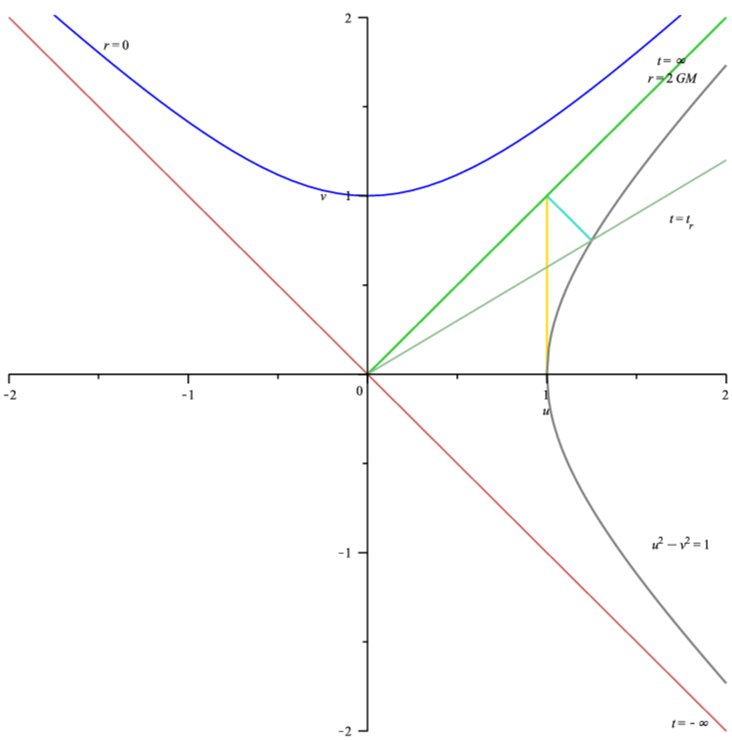
Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 15; Problems 15.7, 15.9.

In this example (Moore, problem P15.9), we have a spaceship maintaining a constant distance R from a black hole, such that the Kruskal-Szekeres (KS) coordinates satisfy

$$u^2 - v^2 = \left(\frac{R}{2GM} - 1\right) e^{R/2GM} = 1$$

At Schwarzschild (S) time $t = 0$ (corresponding to $v = 0$), a shuttle leaves the spaceship and gets pulled towards the black hole along the line $u = 1$ in the KS diagram. If the mother ship is capable of speeds up to light speed, what is the latest time that it can leave its orbit to intercept the shuttle before the shuttle crosses the event horizon?



In the diagram, the shuttle's world line is shown in yellow and the ship's world line is the grey hyperbola. If the ship suddenly breaks out of its orbit and travels at light speed towards the black hole, and intercepts the shuttle just before it crosses the event horizon, the ship will have to follow the turquoise diagonal. (Remember that photon world lines have slopes of ± 1 on a KS diagram.) We therefore need to find the intersection of the turquoise line and the grey hyperbola, and then find the S time t_r at which the ship starts on its rescue mission. On a KS diagram, curves of constant t are straight lines through the origin, so the time is the thin green line in the diagram.

The turquoise line has slope -1 and passes through the point $(1,1)$ so its equation is

$$\begin{aligned} v-1 &= -(u-1) \\ v &= -u+2 \end{aligned}$$

The intersection with the hyperbola is

$$\begin{aligned} u^2-(-u+2)^2 &= 1 \\ 4u-4 &= 1 \\ u &= \frac{5}{4} \\ v &= \frac{3}{4} \end{aligned}$$

The S time is

$$\begin{aligned} t_r &= 2GM\ln\frac{u+v}{u-v} \\ &= 2GM\ln\frac{8}{2} \\ &= 4GM\ln 2 \\ &\approx 2.77GM \end{aligned}$$

As a slight variant on this problem (Moore, problem P15.7), suppose you are working on the outside of the spaceship hovering at radius R and you drop a valuable piece of equipment, which then falls towards the black hole. Even though an observer at infinity would say that the object gets stuck at the event horizon (since the S t coordinate becomes infinite there), the KS diagram shows that you have a limited time (as measured by your own proper time) to go after the object if you are to catch it before it crosses the horizon. In this case, the dropped object has a world line similar to the yellow line on the diagram (not quite the same line, since the dropped object would follow a geodesic, which isn't a straight line here), and to catch it you'd have to leave your orbit at light speed along a turquoise diagonal that intercepts the object just before it crosses the horizon.