

BLACK HOLE RADIATION: ENERGY AT INFINITY OF RADIATED PARTICLE

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 16; Box 16.2.

In radiation from a black hole, we found an estimate for the energy of a radiated particle as measured in the frame of an observer who is momentarily at rest at the point at which the particle pair is created.

$$(1) \quad E = \frac{\hbar}{2\sqrt{2GM\epsilon}}$$

To work out the energy at infinity in the Schwarzschild (S) frame, we can use the particle's four-momentum in the form

$$(2) \quad E = -\mathbf{o}_t \cdot \mathbf{p}$$

where \mathbf{o}_t is the time basis vector and \mathbf{p} is the four-momentum of the particle in the observer's frame. In this frame, $\mathbf{o}_t = [1, 0, 0, 0]$ and E is the time component of \mathbf{p} so 2 follows.

If we work out this equation in S coordinates, we have

$$(3) \quad \mathbf{o}_t = \left[\left(1 - \frac{2GM}{r}\right)^{-1/2}, 0, 0, 0 \right]$$

so using the S metric, we have

$$(4) \quad E = -g_{ij}o_t^i p^j$$

$$(5) \quad = \left(1 - \frac{2GM}{r}\right) \left(1 - \frac{2GM}{r}\right)^{-1/2} p^t$$

$$(6) \quad = \sqrt{1 - \frac{2GM}{r}} m \frac{dt}{d\tau}$$

Since the energy per unit mass of a particle is given by

$$(7) \quad e = \left(1 - \frac{2GM}{r}\right) \frac{dt}{d\tau}$$

energy per unit mass of a particle
we get

$$(8) \quad E = \frac{me}{\sqrt{1 - 2GM/r}}$$

Since e is the energy per unit mass at infinity, $me = E_\infty$ is the total energy of the particle at infinity, so

$$(9) \quad E_\infty = \sqrt{1 - \frac{2GM}{r}} E$$

For a particle that is created at $r = 2GM + \varepsilon$ with energy given by 1, the energy at infinity is

$$(10) \quad E_\infty = \sqrt{1 - \frac{2GM}{2GM + \varepsilon}} \frac{\hbar}{2\sqrt{2GM\varepsilon}}$$

This can be expanded in a series around $\varepsilon = 0$ to get

$$(11) \quad E_\infty = \frac{\hbar}{4GM} - \frac{\hbar}{(4GM)^2} \varepsilon + \frac{3}{2} \frac{\hbar}{(4GM)^3} \varepsilon^2 + \dots$$

Thus for very small distances from the event horizon, the energy the particle has at infinity tends to a constant.

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