BLACK HOLE RADIATION: ENERGY AT INFINITY OF RADIATED PARTICLE

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 16; Box 16.2.

In radiation from a black hole, we found an estimate for the energy of a radiated particle as measured in the frame of an observer who is momentarily at rest at the point at which the particle pair is created.

$$E=\frac{\hbar}{2\sqrt{2GM\varepsilon}} \tag{1}$$

To work out the energy at infinity in the Schwarzschild (S) frame, we can use the particle's four-momentum in the form

$$E=-\mathbf{o}_{t}\cdot\mathbf{p} \tag{2}$$

where $\mathbf{o}_t$ is the time basis vector and $\mathbf{p}$ is the four-momentum of the particle in the observer's frame. In this frame, $\mathbf{o}_t=[1,0,0,0]$ and E is the time component of $\mathbf{p}$ so 2 follows.

If we work out this equation in S coordinates, we have

$$\mathbf{o}_{t}=\left[\left(1-\frac{2GM}{r}\right)^{-1/2},0,0,0\right] \tag{3}$$

so using the S metric, we have

$$\begin{aligned}
E &= -g_{ij}o_{t}^{i}p^{j} && (4)\\
&= \left(1-\frac{2GM}{r}\right)\left(1-\frac{2GM}{r}\right)^{-1/2}p^{t} && (5)\\
&= \sqrt{1-\frac{2GM}{r}}m\frac{dt}{d\tau} && (6)
\end{aligned}$$

Since the energy per unit mass of a particle is given by

$$e=\left(1-\frac{2GM}{r}\right)\frac{dt}{d\tau} \tag{7}$$

energy per unit mass of a particle
we get

$$E=\frac{me}{\sqrt{1-2GM/r}} \tag{8}$$

Since e is the energy per unit mass at infinity, $me=E_{\infty}$ is the total energy of the particle at infinity, so

$$E_{\infty}=\sqrt{1-\frac{2GM}{r}}E \tag{9}$$

For a particle that is created at $r=2GM+\varepsilon$ with energy given by 1, the energy at infinity is

$$E_{\infty}=\sqrt{1-\frac{2GM}{2GM+\varepsilon}}\frac{\hbar}{2\sqrt{2GM\varepsilon}} \tag{10}$$

This can be expanded in a series around $\varepsilon=0$ to get

$$E_{\infty}=\frac{\hbar}{4GM}-\frac{\hbar}{(4GM)^{2}}\varepsilon+\frac{3}{2}\frac{\hbar}{(4GM)^{3}}\varepsilon^{2}+\ldots \tag{11}$$

Thus for very small distances from the event horizon, the energy the particle has at infinity tends to a constant.

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