BLACK HOLE RADIATION: ENERGY AT INFINITY OF RADIATED PARTICLE

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 16; Box 16.2.

In radiation from a black hole, we found an estimate for the energy of a radiated particle as measured in the frame of an observer who is momentarily at rest at the point at which the particle pair is created.

$$E=\frac{\hbar}{2\sqrt{2GM\varepsilon}} \tag{0.1}$$

To work out the energy at infinity in the Schwarzschild (S) frame, we can use the particle's four-momentum in the form

$$E=-\mathbf{o}_t\cdot\mathbf{p} \tag{0.2}$$

where $\mathbf{o}_t$ is the time basis vector and $\mathbf{p}$ is the four-momentum of the particle in the observer's frame. In this frame, $\mathbf{o}_t=[1,0,0,0]$ and E is the time component of $\mathbf{p}$ so 0.2 follows.

If we work out this equation in S coordinates, we have

$$\mathbf{o}_t=\left[\left(1-\frac{2GM}{r}\right)^{-1/2},0,0,0\right] \tag{0.3}$$

so using the S metric, we have

$$\begin{aligned}
E &= -g_{ij}o_t^ip^j && (0.4)\\
&= \left(1-\frac{2GM}{r}\right)\left(1-\frac{2GM}{r}\right)^{-1/2}p^t && (0.5)\\
&= \sqrt{1-\frac{2GM}{r}}m\frac{dt}{d\tau} && (0.6)
\end{aligned}$$

Since the energy per unit mass of a particle is given by

$$e=\left(1-\frac{2GM}{r}\right)\frac{dt}{d\tau} \tag{0.7}$$

energy per unit mass of a particle
we get

$$E=\frac{me}{\sqrt{1-2GM/r}} \tag{0.8}$$

Since e is the energy per unit mass at infinity, $me=E_\infty$ is the total energy of the particle at infinity, so

$$E_\infty=\sqrt{1-\frac{2GM}{r}}E \tag{0.9}$$

For a particle that is created at $r=2GM+\varepsilon$ with energy given by 0.1, the energy at infinity is

$$E_\infty=\sqrt{1-\frac{2GM}{2GM+\varepsilon}}\frac{\hbar}{2\sqrt{2GM\varepsilon}} \tag{0.10}$$

This can be expanded in a series around $\varepsilon=0$ to get

$$E_\infty=\frac{\hbar}{4GM}-\frac{\hbar}{(4GM)^2}\varepsilon+\frac{3}{2}\frac{\hbar}{(4GM)^3}\varepsilon^2+\ldots \tag{0.11}$$

Thus for very small distances from the event horizon, the energy the particle has at infinity tends to a constant.

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