BLACK HOLE EVAPORATION: REMNANTS OF THE BIG BANG

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 16; Problem P16.2.

We've seen that through radiation, a black hole can eventually evaporate in a time t_0 :

$$\boxed{t_0 = 2.0903 \times 10^{67} \left(\frac{M}{M_s}\right)^3 \text{ years}} \tag{1}$$

where M_s is the solar mass. Clearly if we're going to observe black hole evaporation, its mass M must be considerably less than the sun's. Suppose black holes were formed during the big bang at 13.7×10^9 years ago. If the black hole is just evaporating now, its mass would have been:

$$\begin{aligned}
M &= M_s \left(\frac{13.7 \times 10^9}{2.0903 \times 10^{67}}\right)^{1/3} && (2)\\
&= 8.69 \times 10^{-20} M_s && (3)\\
&= \left(8.69 \times 10^{-20}\right)\left(1.989 \times 10^{30}\right) \text{ kg} && (4)\\
&= 1.728 \times 10^{11} \text{ kg} && (5)
\end{aligned}$$

To put this in perspective, this is equivalent to an asteroid, with a typical rocky density of $\rho = 5000$ kg m^{-3}, with a radius of

$$\begin{aligned}
R &= \left(\frac{3M}{4\pi\rho}\right)^{1/3} && (6)\\
&= 202 \text{ m} && (7)
\end{aligned}$$

To see how much energy is released in the final second of the black hole's life, we can start with the equation we had earlier from the Stefan-Boltzmann relation:

$$M^2 dM = -\frac{\sigma\hbar^4}{256\pi^3 k_B^4 G^2} dt \tag{8}$$

If we integrate this from $t=0$ to a time t_1 one second before the present, at which time the black hole's remaining mass is M_1, then

$$\int_{M_0}^{M_1} M^2 dM = -\frac{\sigma\hbar^4}{256\pi^3 k_B^4 G^2}\int_0^{t_1} dt \tag{9}$$

$$\frac{1}{3}\left(M_1^3 - M_0^3\right) = -\frac{\sigma\hbar^4}{256\pi^3 k_B^4 G^2} t_1 \tag{10}$$

$$M_1 = \left[M_0^3 - \frac{3\sigma\hbar^4}{256\pi^3 k_B^4 G^2} t_1\right]^{1/3} \tag{11}$$

However, M_0^3 is just $\frac{3\sigma\hbar^4}{256\pi^3 k_B^4 G^2} t_p$, where t_p is the present time, so that $t_p - t_1 = 1$ s. Therefore the mass remaining 1 second before complete evaporation is:

$$M_1 = \left[\frac{3\sigma\hbar^4}{256\pi^3 k_B^4 G^2}\left(t_p - t_1\right)\right]^{1/3} \tag{12}$$

$$= 341.38\left(t_p - t_1\right)^{1/3} \tag{13}$$

In GR units, a time of 1 second is 3×10^8 m so we get for the mass

$$M_1 = 2.28\times10^5 \text{ kg} \tag{14}$$

This is equivalent to

$$E = M_1 c^2 = 2.06\times10^{22} \text{ J} \tag{15}$$

which is about 500,000 times the energy released in an atomic bomb blast.

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