

BLACK HOLE EVAPORATION: REMNANTS OF THE BIG BANG

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 16; Problem P16.2.

We've seen that through radiation, a black hole can eventually evaporate in a time t_0 :

$$(1) \quad t_0 = 2.0903 \times 10^{67} \left(\frac{M}{M_s} \right)^3 \text{ years}$$

where M_s is the solar mass. Clearly if we're going to observe black hole evaporation, its mass M must be considerably less than the sun's. Suppose black holes were formed during the big bang at 13.7×10^9 years ago. If the black hole is just evaporating now, its mass would have been:

$$(2) \quad M = M_s \left(\frac{13.7 \times 10^9}{2.0903 \times 10^{67}} \right)^{1/3}$$

$$(3) \quad = 8.69 \times 10^{-20} M_s$$

$$(4) \quad = (8.69 \times 10^{-20}) (1.989 \times 10^{30}) \text{ kg}$$

$$(5) \quad = 1.728 \times 10^{11} \text{ kg}$$

To put this in perspective, this is equivalent to an asteroid, with a typical rocky density of $\rho = 5000 \text{ kg m}^{-3}$, with a radius of

$$(6) \quad R = \left(\frac{3M}{4\pi\rho} \right)^{1/3}$$

$$(7) \quad = 202 \text{ m}$$

To see how much energy is released in the final second of the black hole's life, we can start with the equation we had earlier from the Stefan-Boltzmann relation:

$$(8) \quad M^2 dM = - \frac{\sigma \hbar^4}{256\pi^3 k_B^4 G^2} dt$$

If we integrate this from $t = 0$ to a time t_1 one second before the present, at which time the black hole's remaining mass is M_1 , then

$$(9) \quad \int_{M_0}^{M_1} M^2 dM = -\frac{\sigma \hbar^4}{256\pi^3 k_B^4 G^2} \int_0^{t_1} dt$$

$$(10) \quad \frac{1}{3} (M_1^3 - M_0^3) = -\frac{\sigma \hbar^4}{256\pi^3 k_B^4 G^2} t_1$$

$$(11) \quad M_1 = \left[M_0^3 - \frac{3\sigma \hbar^4}{256\pi^3 k_B^4 G^2} t_1 \right]^{1/3}$$

However, M_0^3 is just $\frac{3\sigma \hbar^4}{256\pi^3 k_B^4 G^2} t_p$, where t_p is the present time, so that $t_p - t_1 = 1$ s. Therefore the mass remaining 1 second before complete evaporation is:

$$(12) \quad M_1 = \left[\frac{3\sigma \hbar^4}{256\pi^3 k_B^4 G^2} (t_p - t_1) \right]^{1/3}$$

$$(13) \quad = 341.38 (t_p - t_1)^{1/3}$$

In GR units, a time of 1 second is 3×10^8 m so we get for the mass

$$(14) \quad M_1 = 2.28 \times 10^5 \text{ kg}$$

This is equivalent to

$$(15) \quad E = M_1 c^2 = 2.06 \times 10^{22} \text{ J}$$

which is about 500,000 times the energy released in an atomic bomb blast.

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