

BLACK HOLE EVAPORATION: HOW LONG WILL A BLACK HOLE LIVE?

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 16; Boxes 16.3, 16.4.

Since a black hole can radiate its energy away and the energy of a black hole is equivalent to its mass, it's possible that a black hole will simply evaporate after a long enough time. We can get an estimate of this time as follows. First, we recall that, for particle-antiparticle pairs that are produced very close to the event horizon, the energy of the particle at infinity tends to a constant:

$$(0.1) \quad E_\infty = \frac{\hbar}{4GM}$$

This is only a crude estimate based on oversimplifying the situation and a more accurate calculation, due to Stephen Hawking, results in

$$(0.2) \quad E_\infty = \frac{\hbar}{8\pi GM}$$

In thermodynamics, a *blackbody* is a body that absorbs all frequencies of electromagnetic radiation (hence 'black') and radiates with a spectrum that depends only its temperature T . The wavelength of radiation peaks at a shorter value the higher the temperature (which is why an iron bar, for example, glows red when it gets hotter; at cooler temperatures it radiates in the infra-red). Hawking showed that the energy spectrum of a black hole is actually the same as that of a blackbody with temperature T if we set $E_\infty = k_B T$, where k_B is Boltzmann's constant. We can therefore define the temperature of a black hole as

$$(0.3) \quad T = \frac{\hbar}{8\pi k_B GM}$$

To use this in calculations, it's best to convert the constants to relativistic form, where $c = 1$. We start with their values in SI units. For Planck's constant:

$$(0.4) \quad \hbar = 1.0546 \times 10^{-34} \text{ J s}$$

$$(0.5) \quad = 1.0546 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1} \times (3 \times 10^8)^{-1} \text{ s m}^{-1}$$

$$(0.6) \quad = 3.5153 \times 10^{-43} \text{ kg m}$$

For Boltzmann's constant:

$$(0.7) \quad k_B = 1.3807 \times 10^{-23} \text{ J K}^{-1}$$

$$(0.8) \quad = 1.3807 \times 10^{-23} \text{ kg m}^2 \text{ s}^{-2} \text{ K}^{-1} \times (3 \times 10^8)^{-2} \text{ s}^2 \text{ m}^{-2}$$

$$(0.9) \quad = 1.534 \times 10^{-40} \text{ kg K}^{-1}$$

Thus the temperature is

$$(0.10) \quad T = \frac{9.118 \times 10^{-5}}{GM} \text{ K}$$

In terms of the solar mass M_s , we can use $GM_s = 1477 \text{ m}$ so

$$(0.11) \quad T = \frac{9.118 \times 10^{-5} M_s}{GM_s} \frac{M_s}{M} \text{ K}$$

$$(0.12) \quad = 6.173 \times 10^{-8} \frac{M_s}{M} \text{ K}$$

A one solar mass black hole is thus almost at absolute zero.

The rate at which an object radiates energy is given by the Stefan-Boltzmann law, which states

$$(0.13) \quad \frac{dE}{dt} = A\sigma T^4$$

where A is the surface area of the object and $\sigma = 2.105 \times 10^{-33} \text{ kg m}^{-3} \text{ K}^{-4}$ is the Stefan-Boltzmann constant, in relativistic units. In the case of a black hole, the energy is just the mass, so

$$(0.14) \quad \frac{dE}{dt} = -\frac{dM}{dt}$$

and the area is that of a sphere with a radius $r = 2GM$, so

$$(0.15) \quad A = 4\pi(2GM)^2 = 16\pi(GM)^2$$

Using 0.3, we then have

$$(0.16) \quad \frac{dM}{dt} = -16\pi(GM)^2 \sigma \left(\frac{\hbar}{8\pi k_B GM} \right)^4$$

$$(0.17) \quad = -\frac{\sigma \hbar^4}{256\pi^3 k_B^4 G^2} \frac{1}{M^2}$$

$$(0.18) \quad M^2 dM = -\frac{\sigma \hbar^4}{256\pi^3 k_B^4 G^2} dt$$

$$(0.19) \quad \frac{1}{3} M^3 = \frac{\sigma \hbar^4}{256\pi^3 k_B^4 G^2} \Delta t$$

$$(0.20) \quad \Delta t = \frac{256\pi^3 k_B^4}{3\sigma \hbar^4 G} (GM)^3$$

$$(0.21) \quad = \frac{256\pi^3 k_B^4}{3\sigma \hbar^4 G} \left(\frac{GM}{GM_s} \right)^3 (GM_s)^3$$

where Δt is the time required for the black hole to evaporate completely. We can plug in the numbers, using $G = 7.426 \times 10^{-28} \text{ m kg}^{-1}$ and $1 \text{ year} = 9.461 \times 10^{15} \text{ m}$ to get

$$(0.22) \quad \Delta t = 1.9777 \times 10^{83} \left(\frac{M}{M_s} \right)^3 \text{ m}$$

$$(0.23) \quad = 2.0903 \times 10^{67} \left(\frac{M}{M_s} \right)^3 \text{ years}$$

In other words, for a one solar mass black hole, we're not going to see it evaporate any time soon.

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