

BLACK HOLE EVAPORATION: HOW LONG WILL A BLACK HOLE LIVE?

Link to: [physicspages home page](#).

To leave a comment or report an error, please use the auxiliary blog.

Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 16; Boxes 16.3, 16.4.

Since a black hole can radiate its energy away and the energy of a black hole is equivalent to its mass, it's possible that a black hole will simply evaporate after a long enough time. We can get an estimate of this time as follows. First, we recall that, for particle-antiparticle pairs that are produced very close to the event horizon, the energy of the particle at infinity tends to a constant:

$$E_{\infty} = \frac{\hbar}{4GM} \quad (1)$$

This is only a crude estimate based on oversimplifying the situation and a more accurate calculation, due to Stephen Hawking, results in

$$E_{\infty} = \frac{\hbar}{8\pi GM} \quad (2)$$

In thermodynamics, a *blackbody* is a body that absorbs all frequencies of electromagnetic radiation (hence 'black') and radiates with a spectrum that depends only its temperature T . The wavelength of radiation peaks at a shorter value the higher the temperature (which is why an iron bar, for example, glows red when it gets hotter; at cooler temperatures it radiates in the infra-red). Hawking showed that the energy spectrum of a black hole is actually the same as that of a blackbody with temperature T if we set $E_{\infty} = k_B T$, where k_B is Boltzmann's constant. We can therefore define the temperature of a black hole as

$$T = \frac{\hbar}{8\pi k_B GM} \quad (3)$$

To use this in calculations, it's best to convert the constants to relativistic form, where $c = 1$. We start with their values in SI units. For Planck's constant:

$$\hbar = 1.0546 \times 10^{-34} \text{ J s} \quad (4)$$

$$= 1.0546 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1} \times (3 \times 10^8)^{-1} \text{ s m}^{-1} \quad (5)$$

$$= 3.5153 \times 10^{-43} \text{ kg m} \quad (6)$$

For Boltzmann's constant:

$$k_B = 1.3807 \times 10^{-23} \text{ J K}^{-1} \quad (7)$$

$$= 1.3807 \times 10^{-23} \text{ kg m}^2 \text{ s}^{-2} \text{ K}^{-1} \times (3 \times 10^8)^{-2} \text{ s}^2 \text{ m}^{-2} \quad (8)$$

$$= 1.534 \times 10^{-40} \text{ kg K}^{-1} \quad (9)$$

Thus the temperature is

$$T = \frac{9.118 \times 10^{-5}}{GM} \text{ K} \quad (10)$$

In terms of the solar mass M_s , we can use $GM_s = 1477 \text{ m}$ so

$$T = \frac{9.118 \times 10^{-5} M_s}{GM_s M} \text{ K} \quad (11)$$

$$= 6.173 \times 10^{-8} \frac{M_s}{M} \text{ K} \quad (12)$$

A one solar mass black hole is thus almost at absolute zero.

The rate at which an object radiates energy is given by the Stefan-Boltzmann law, which states

$$\frac{dE}{dt} = A\sigma T^4 \quad (13)$$

where A is the surface area of the object and $\sigma = 2.105 \times 10^{-33} \text{ kg m}^{-3} \text{ K}^{-4}$ is the Stefan-Boltzmann constant, in relativistic units. In the case of a black hole, the energy is just the mass, so

$$\frac{dE}{dt} = -\frac{dM}{dt} \quad (14)$$

and the area is that of a sphere with a radius $r = 2GM$, so

$$A = 4\pi (2GM)^2 = 16\pi (GM)^2 \quad (15)$$

Using 3, we then have

$$\frac{dM}{dt} = -16\pi (GM)^2 \sigma \left(\frac{\hbar}{8\pi k_B GM} \right)^4 \quad (16)$$

$$= -\frac{\sigma \hbar^4}{256\pi^3 k_B^4 G^2} \frac{1}{M^2} \quad (17)$$

$$M^2 dM = -\frac{\sigma \hbar^4}{256\pi^3 k_B^4 G^2} dt \quad (18)$$

$$\frac{1}{3} M^3 = \frac{\sigma \hbar^4}{256\pi^3 k_B^4 G^2} \Delta t \quad (19)$$

$$\Delta t = \frac{256\pi^3 k_B^4}{3\sigma \hbar^4 G} (GM)^3 \quad (20)$$

$$= \frac{256\pi^3 k_B^4}{3\sigma \hbar^4 G} \left(\frac{GM}{GM_s} \right)^3 (GM_s)^3 \quad (21)$$

where Δt is the time required for the black hole to evaporate completely. We can plug in the numbers, using $G = 7.426 \times 10^{-28} \text{ m kg}^{-1}$ and 1 year = $9.461 \times 10^{15} \text{ m}$ to get

$$\Delta t = 1.9777 \times 10^{83} \left(\frac{M}{M_s} \right)^3 \text{ m} \quad (22)$$

$$= 2.0903 \times 10^{67} \left(\frac{M}{M_s} \right)^3 \text{ years} \quad (23)$$

In other words, for a one solar mass black hole, we're not going to see it evaporate any time soon.

PINGBACKS

Pingback: Black hole radiation: energy of a particle from a solar mass black hole

Pingback: Black hole evaporation: remnants of the big bang

Pingback: Black holes and the Large Hadron Collider

Pingback: Black hole heat engine

Pingback: Black hole temperatures at different distances

Pingback: Black hole radiation: mass as a function of time

Pingback: Black hole entropy

Pingback: Black hole entropy: it's pretty large

Pingback: Temperature of a black hole