BLACK HOLE EVAPORATION: HOW LONG WILL A BLACK HOLE LIVE?

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 16; Boxes 16.3, 16.4.

Since a black hole can radiate its energy away and the energy of a black hole is equivalent to its mass, it's possible that a black hole will simply evaporate after a long enough time. We can get an estimate of this time as follows. First, we recall that, for particle-antiparticle pairs that are produced very close to the event horizon, the energy of the particle at infinity tends to a constant:

$$E_{\infty}=\frac{\hbar}{4GM} \tag{1}$$

This is only a crude estimate based on oversimplifying the situation and a more accurate calculation, due to Stephen Hawking, results in

$$E_{\infty}=\frac{\hbar}{8\pi GM} \tag{2}$$

In thermodynamics, a *blackbody* is a body that absorbs all frequencies of electromagnetic radiation (hence 'black') and radiates with a spectrum that depends only its temperature T. The wavelength of radiation peaks at a shorter value the higher the temperature (which is why an iron bar, for example, glows red when it gets hotter; at cooler temperatures it radiates in the infra-red). Hawking showed that the energy spectrum of a black hole is actually the same as that of a blackbody with temperature T if we set $E_{\infty}=k_BT$, where k_B is Boltzmann's constant. We can therefore define the temperature of a black hole as

$$\boxed{T=\frac{\hbar}{8\pi k_BGM}} \tag{3}$$

To use this in calculations, it's best to convert the constants to relativistic form, where $c=1$. We start with their values in SI units. For Planck's constant:

$$\begin{aligned}
\hbar &= 1.0546\times10^{-34}\text{ J s} && (4)\\
&= 1.0546\times10^{-34}\text{ kg m}^2\text{ s}^{-1}\times\left(3\times10^{8}\right)^{-1}\text{ s m}^{-1} && (5)\\
&= 3.5153\times10^{-43}\text{ kg m} && (6)
\end{aligned}$$

For Boltzmann's constant:

$$\begin{aligned}
k_B &= 1.3807\times10^{-23}\text{ J K}^{-1} && (7)\\
&= 1.3807\times10^{-23}\text{ kg m}^2\text{ s}^{-2}\text{ K}^{-1}\times\left(3\times10^{8}\right)^{-2}\text{ s}^2\text{ m}^{-2} && (8)\\
&= 1.534\times10^{-40}\text{ kg K}^{-1} && (9)
\end{aligned}$$

Thus the temperature is

$$T=\frac{9.118\times10^{-5}}{GM}\text{ K} \tag{10}$$

In terms of the solar mass M_s, we can use $GM_s = 1477$ m so

$$\begin{aligned}
T &= \frac{9.118\times10^{-5}}{GM_s}\frac{M_s}{M}\text{ K} && (11)\\
&= 6.173\times10^{-8}\frac{M_s}{M}\text{ K} && (12)
\end{aligned}$$

A one solar mass black hole is thus almost at absolute zero.

The rate at which an object radiates energy is given by the Stefan-Boltzmann law, which states

$$\frac{dE}{dt}=A\sigma T^4 \tag{13}$$

where A is the surface area of the object and $\sigma = 2.105\times10^{-33}$ kg m^{-3} K^{-4} is the Stefan-Boltzmann constant, in relativistic units. In the case of a black hole, the energy is just the mass, so

$$\frac{dE}{dt}=-\frac{dM}{dt} \tag{14}$$

and the area is that of a sphere with a radius $r = 2GM$, so

$$A=4\pi(2GM)^2=16\pi(GM)^2 \tag{15}$$

Using 3, we then have

$$\frac{dM}{dt} = -16\pi (GM)^2 \sigma \left(\frac{\hbar}{8\pi k_B GM}\right)^4 \tag{16}$$

$$= -\frac{\sigma\hbar^4}{256\pi^3 k_B^4 G^2}\frac{1}{M^2} \tag{17}$$

$$M^2 dM = -\frac{\sigma\hbar^4}{256\pi^3 k_B^4 G^2} dt \tag{18}$$

$$\frac{1}{3}M^3 = \frac{\sigma\hbar^4}{256\pi^3 k_B^4 G^2}\Delta t \tag{19}$$

$$\Delta t = \frac{256\pi^3 k_B^4}{3\sigma\hbar^4 G}(GM)^3 \tag{20}$$

$$= \frac{256\pi^3 k_B^4}{3\sigma\hbar^4 G}\left(\frac{GM}{GM_s}\right)^3 (GM_s)^3 \tag{21}$$

where Δt is the time required for the black hole to evaporate completely. We can plug in the numbers, using $G = 7.426 \times 10^{-28}$ m kg^{-1} and 1 year $= 9.461 \times 10^{15}$ m to get

$$\Delta t = 1.9777 \times 10^{83} \left(\frac{M}{M_s}\right)^3 \text{ m} \tag{22}$$

$$= 2.0903 \times 10^{67} \left(\frac{M}{M_s}\right)^3 \text{ years} \tag{23}$$

In other words, for a one solar mass black hole, we're not going to see it evaporate any time soon.

Pingbacks

Pingback: Black hole radiation: energy of a particle from a solar mass black hole
Pingback: Black hole evaporation: remnants of the big bang
Pingback: Black holes and the Large Hadron Collider
Pingback: Black hole heat engine
Pingback: Black hole temperatures at different distances
Pingback: Black hole radiation: mass as a function of time
Pingback: Black hole entropy
Pingback: Black hole entropy: it's pretty large
Pingback: Temperature of a black hole