BLACK HOLE EVAPORATION: HOW LONG WILL A BLACK HOLE LIVE?

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 16; Boxes 16.3, 16.4.

Since a black hole can radiate its energy away and the energy of a black hole is equivalent to its mass, it's possible that a black hole will simply evaporate after a long enough time. We can get an estimate of this time as follows. First, we recall that, for particle-antiparticle pairs that are produced very close to the event horizon, the energy of the particle at infinity tends to a constant:

$$E_{\infty}=\frac{\hbar}{4GM} \tag{1}$$

This is only a crude estimate based on oversimplifying the situation and a more accurate calculation, due to Stephen Hawking, results in

$$E_{\infty}=\frac{\hbar}{8\pi GM} \tag{2}$$

In thermodynamics, a *blackbody* is a body that absorbs all frequencies of electromagnetic radiation (hence 'black') and radiates with a spectrum that depends only its temperature T. The wavelength of radiation peaks at a shorter value the higher the temperature (which is why an iron bar, for example, glows red when it gets hotter; at cooler temperatures it radiates in the infra-red). Hawking showed that the energy spectrum of a black hole is actually the same as that of a blackbody with temperature T if we set $E_{\infty}=k_BT$, where k_B is Boltzmann's constant. We can therefore define the temperature of a black hole as

$$\boxed{T=\frac{\hbar}{8\pi k_BGM}} \tag{3}$$

To use this in calculations, it's best to convert the constants to relativistic form, where $c=1$. We start with their values in SI units. For Planck's constant:

$$\hbar = 1.0546 \times 10^{-34} \text{ J s} \tag{4}$$
$$= 1.0546 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1} \times \left(3 \times 10^8\right)^{-1} \text{ s m}^{-1} \tag{5}$$
$$= 3.5153 \times 10^{-43} \text{ kg m} \tag{6}$$

For Boltzmann's constant:

$$k_B = 1.3807 \times 10^{-23} \text{ J K}^{-1} \tag{7}$$
$$= 1.3807 \times 10^{-23} \text{ kg m}^2 \text{ s}^{-2} \text{ K}^{-1} \times \left(3 \times 10^8\right)^{-2} \text{ s}^2 \text{ m}^{-2} \tag{8}$$
$$= 1.534 \times 10^{-40} \text{ kg K}^{-1} \tag{9}$$

Thus the temperature is

$$T = \frac{9.118 \times 10^{-5}}{GM} \text{ K} \tag{10}$$

In terms of the solar mass M_s, we can use $GM_s = 1477$ m so

$$T = \frac{9.118 \times 10^{-5}}{GM_s} \frac{M_s}{M} \text{ K} \tag{11}$$
$$= 6.173 \times 10^{-8} \frac{M_s}{M} \text{ K} \tag{12}$$

A one solar mass black hole is thus almost at absolute zero.

The rate at which an object radiates energy is given by the Stefan-Boltzmann law, which states

$$\frac{dE}{dt} = A\sigma T^4 \tag{13}$$

where A is the surface area of the object and $\sigma = 2.105 \times 10^{-33}$ kg m^{-3} K^{-4} is the Stefan-Boltzmann constant, in relativistic units. In the case of a black hole, the energy is just the mass, so

$$\frac{dE}{dt} = -\frac{dM}{dt} \tag{14}$$

and the area is that of a sphere with a radius $r = 2GM$, so

$$A = 4\pi (2GM)^2 = 16\pi (GM)^2 \tag{15}$$

Using 3, we then have

$$\frac{dM}{dt} = -16\pi(GM)^2\sigma\left(\frac{\hbar}{8\pi k_B GM}\right)^4 \tag{16}$$

$$= -\frac{\sigma\hbar^4}{256\pi^3 k_B^4 G^2}\frac{1}{M^2} \tag{17}$$

$$M^2 dM = -\frac{\sigma\hbar^4}{256\pi^3 k_B^4 G^2} dt \tag{18}$$

$$\frac{1}{3}M^3 = \frac{\sigma\hbar^4}{256\pi^3 k_B^4 G^2}\Delta t \tag{19}$$

$$\Delta t = \frac{256\pi^3 k_B^4}{3\sigma\hbar^4 G}(GM)^3 \tag{20}$$

$$= \frac{256\pi^3 k_B^4}{3\sigma\hbar^4 G}\left(\frac{GM}{GM_s}\right)^3 (GM_s)^3 \tag{21}$$

where Δt is the time required for the black hole to evaporate completely. We can plug in the numbers, using $G = 7.426 \times 10^{-28}$ m kg^{-1} and 1 year $= 9.461 \times 10^{15}$ m to get

$$\Delta t = 1.9777 \times 10^{83}\left(\frac{M}{M_s}\right)^3 \text{ m} \tag{22}$$

$$= 2.0903 \times 10^{67}\left(\frac{M}{M_s}\right)^3 \text{ years} \tag{23}$$

In other words, for a one solar mass black hole, we're not going to see it evaporate any time soon.

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