BLACK HOLE TEMPERATURES AT DIFFERENT DISTANCES

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 16; Problem P16.5.

We've seen that a temperature can be defined for a black hole that radiates particles (mainly photons) due to quantum pair creation:

$$T_{\infty}=\frac{\hbar}{8\pi k_{B}GM} \tag{1}$$

This is the temperature as seen by an observer at infinity. If we take the temperature to be proportional to the observed energy of the emitted particles, we can get the temperature as measured by an observer at rest at some finite distance R from the black hole. The energy of the photons is

$$E=\frac{E_{\infty}}{\sqrt{1-2GM/R}} \tag{2}$$

where E_{∞} is the energy at infinity. Using the proportionality of T and E, we get

$$T=\frac{T_{\infty}}{\sqrt{1-2GM/R}} \tag{3}$$

The temperature thus becomes infinite at $R=2GM$. For a solar mass black hole, we have

$$T_{\infty}=6.173\times10^{-8}\text{ K} \tag{4}$$

We have to get extraordinarily close to a black hole to see the temperature rise significantly. Solving 3 for $R(T)$ we get

$$R(T)=2GM\left(1+\frac{T_{\infty}^{2}}{T^{2}-T_{\infty}^{2}}\right) \tag{5}$$

For example, if we want $T=300$ K (around room temperature), then

$$R(300)=2GM\left(1+4.23\times10^{-20}\right) \tag{6}$$

Since $GM=1477$ m, we need to approach to within 1.25×10^{-16} m of the event horizon. This is smaller than the radius of a proton (around 8.8×10^{-16} m).