

## BLACK HOLE ENTROPY

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 16; Problem P16.7.

Bizarre as it seems, an entropy can be defined for a black hole. From thermodynamics (which we haven't covered yet, so you'll need to look elsewhere for a derivation for now), the entropy  $S$  of a system is defined in terms of its temperature  $T$  and internal energy  $U$  by

$$(1) \quad \boxed{\frac{1}{T} \equiv \frac{\partial S}{\partial U}}$$

A black hole's internal energy is just its mass, so  $U = M$ , and we found an expression for its temperature earlier:

$$(2) \quad T = \frac{\hbar}{8\pi k_B G M}$$

We can thus work out the entropy as a function of mass:

$$(3) \quad \frac{\partial S}{\partial M} = \frac{8\pi k_B G}{\hbar} M$$

$$(4) \quad S = \frac{4\pi k_B G}{\hbar} M^2$$

where the last line assumes that  $S = 0$  at  $M = 0$ . From this, we can see that if we combine two black holes with masses  $M_1$  and  $M_2$ , the total entropy of the system increases.

$$(5) \quad S_{tot} = \frac{4\pi k_B G}{\hbar} (M_1 + M_2)^2$$

$$(6) \quad = \frac{4\pi k_B G}{\hbar} (M_1^2 + M_2^2 + 2M_1 M_2)$$

$$(7) \quad = S_1 + S_2 + \frac{8\pi k_B G}{\hbar} M_1 M_2$$

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