BLACK HOLE ENTROPY

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 16; Problem P16.7.

Bizarre as it seems, an entropy can be defined for a black hole. From thermodynamics (which we haven't covered yet, so you'll need to look elsewhere for a derivation for now), the entropy S of a system is defined in terms of its temperature T and internal energy U by

$$\boxed{\frac{1}{T} \equiv \frac{\partial S}{\partial U}} \tag{1}$$

A black hole's internal energy is just its mass, so $U = M$, and we found an expression for its temperature earlier:

$$T = \frac{\hbar}{8\pi k_B G M} \tag{2}$$

We can thus work out the entropy as a function of mass:

$$\frac{\partial S}{\partial M} = \frac{8\pi k_B G}{\hbar} M \tag{3}$$

$$S = \frac{4\pi k_B G}{\hbar} M^2 \tag{4}$$

where the last line assumes that $S = 0$ at $M = 0$. From this, we can see that if we combine two black holes with masses M_1 and M_2, the total entropy of the system increases.

$$S_{tot} = \frac{4\pi k_B G}{\hbar} (M_1 + M_2)^2 \tag{5}$$

$$= \frac{4\pi k_B G}{\hbar} \left(M_1^2 + M_2^2 + 2M_1 M_2\right) \tag{6}$$

$$= S_1 + S_2 + \frac{8\pi k_B G}{\hbar} M_1 M_2 \tag{7}$$

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