BLACK HOLE IN EQUILIBRIUM WITH A THERMAL RESERVOIR

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 16; Problem P16.8.

The entropy of a black hole is given by

$$S=\frac{4\pi k_{B}G}{\hbar}M^{2} \tag{1}$$

Given that the entropy of a thermal reservoir with temperature T_R and internal energy U (another fact from thermodynamics) is

$$S_{R}=\frac{U}{T_{R}}+C \tag{2}$$

where C is a constant. If a black hole with mass M is exchanging thermal radiation with the reservoir, we can say that the combined (and conserved) total energy of the system is $U_{tot}=U+M$ so

$$S_{tot}=\frac{U_{tot}-M}{T_{R}}+C+\frac{4\pi k_{B}G}{\hbar}M^{2} \tag{3}$$

Also from thermodynamics, it is known that when a system is in thermal equilibrium (no net transfer of radiation between the black hole and the reservoir), then the entropy should be at a local maximum. In this case, we can find the extrema of S_{tot} as a function of M, since the mass of the black hole varies as it exchanges radiation with the reservoir. We get

$$\frac{dS_{tot}}{dM} = -\frac{1}{T_{R}}+\frac{8\pi k_{B}G}{\hbar}M=0 \tag{4}$$

$$M = \frac{\hbar}{8\pi k_{B}GT_{R}} \tag{5}$$

However, from the second derivative:

$$\frac{d^{2}S_{tot}}{dM^{2}}=\frac{8\pi k_{B}G}{\hbar}>0 \tag{6}$$

we see that the entropy is actually at a local *minimum* at this mass and, since the curve 3 is a parabola, the maxima of entropy occur at the two ends

$M = 0$ and $M = U_{tot}$. Which of these two points is the overall maximum depends on U_{tot} and T_R, but it does seem to indicate that equilibrium is achieved only when either the black hole evaporates and sends all its energy (mass) into the reservoir, or else the black hole swallows up the reservoir completely.