BLACK HOLE ENTROPY: IT'S PRETTY LARGE

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 16; Problem P16.9.

The entropy of a solar mass black hole is given by

$$S=\frac{4\pi k_BG}{\hbar}M_s^2 \tag{0.1}$$

How does this compare with the entropy of objects at 'normal' density and room temperature? In the latter case, the entropy is usually around $S_n=Nk_B$ where N is the number of particles making up the object. If we consider a solar mass black hole, and suppose that it is made up exclusively of ionized hydrogen, we can get an estimate of the energy. The number of particles in such an object is

$$N = 2\frac{M_s}{M_H} \tag{0.2}$$

where M_H is the mass of a hydrogen atom, and the factor of 2 accounts for 2 particles (electron + proton) per atom.

The ratio S/Nk_B is then

$$\frac{S}{Nk_B}=\frac{4\pi GM_sM_H}{2\hbar} \tag{0.3}$$

In GR units, this is a dimensionless quantity, since $\hbar$ has dimensions of kg m and G has dimensions of m kg^{-1}. In GR units, the values are

$$\hbar = 3.5153\times10^{-43}\text{ kg m} \tag{0.4}$$

$$G = 7.426\times10^{-28}\text{ m kg}^{-1} \tag{0.5}$$

$$M_s = 1.989\times10^{30}\text{ kg} \tag{0.6}$$

$$M_H = 1.66\times10^{-27}\text{ kg} \tag{0.7}$$

So we get

$$\frac{S}{Nk_B}=4.38\times10^{19} \tag{0.8}$$

The entropy of a black hole is vastly more per particle than in 'normal' matter.