Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 18; Problem P18.2.

Suppose we have an infinite plane of mass. We can look at this as analogous to the infinite plane of charge in electrostatics (see Example 7 here). In Newtonian physics, the gravitational field behaves the same as an electrostatic field, except that mass is always attractive, so we can view the lines of gravitational force as always converging on mass (in electrostatics, this is equivalent to using only negative charges). Since the gravitational and electrical forces are both inverse square, the gravitational field due to an infinite plane of mass is constant (independent of distance from the plane) and points directly towards the plane. Gauss's law for gravity becomes

$$\int_{\mathscr{A}} \mathbf{g}\cdot d\mathbf{a} = 4\pi GM \tag{0.1}$$

where the integral is over a surface $\mathscr{A}$ enclosing a mass M. If the plane has surface mass density σ and we take a cylindrical Gaussian surface of radius a and height $2x$ perpendicular to the plane, then on the RHS of the plane

$$\int_{\mathscr{A}} \mathbf{g}\cdot d\mathbf{a} = -2\pi a^2 g(x) \tag{0.2}$$

$$= 4\pi G\pi a^2 \rho \tag{0.3}$$

$$\mathbf{g} = -2\pi G\rho\hat{\mathbf{x}} \tag{0.4}$$

where the x axis is normal to the plane.

Since the field is constant, the potential must depend linearly on x:

$$\Phi = 2\pi G\rho x \tag{0.5}$$

so its second derivative with respect to any spatial coordinate is zero. From the Newtonian tidal formula:

$$\frac{d^2n^i}{dt^2} = -\eta^{ij}n^k \left.\frac{\partial^2\Phi}{\partial x^k\partial x^j}\right|_{\vec{x}} \tag{0.6}$$

the tidal acceleration is always zero in such a field.

If this result held in general relativity, this result would indicate that space is flat, so the field wouldn't be 'real' in this case. Of course, such a situation shouldn't bother us too much, since infinite planes of mass are in short supply in the real universe.

PINGBACKS

Pingback: Riemann tensor for an infinite plane of mass