

RIEMANN TENSOR IN 2-D FLAT SPACE

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 19; Problem P19.5.

Another example of the Riemann tensor in a 2-d space. The metric is

$$ds^2 = dp^2 + \frac{dq^2}{b^2 q^2} \quad (1)$$

where b is a constant. The metric tensor is therefore $g_{pp} = 1$, $g_{qq} = 1/b^2 q^2$. By comparing the two forms of the geodesic equation, we can calculate the Christoffel symbols.

$$g_{aj} \ddot{x}^j + \left(\partial_i g_{aj} - \frac{1}{2} \partial_a g_{ij} \right) \dot{x}^j \dot{x}^i = 0 \quad (2)$$

$$\ddot{x}^a + \Gamma^a_{ij} \dot{x}^j \dot{x}^i = 0 \quad (3)$$

With $a = p$, we get from 2

$$\ddot{p} = 0 \quad (4)$$

From 3, we see that $\Gamma^p_{ij} = 0$ for all i and j .

With $a = q$, we have

$$\frac{1}{b^2 q^2} \ddot{q} - \frac{2}{b^2 q^3} \dot{q}^2 + \frac{1}{b^2 q^3} \dot{q}^2 = 0 \quad (5)$$

$$\ddot{q} - \frac{1}{q} \dot{q}^2 = 0 \quad (6)$$

Comparing with 3 we find

$$\Gamma^q_{qq} = -\frac{1}{q} \quad (7)$$

$$\Gamma^q_{pq} = \Gamma^q_{qp} = \Gamma^q_{pp} = 0 \quad (8)$$

The only independent component of the Riemann tensor in 2-d is R^p_{qpq} :

$$R_{ppq}^p = \partial_p \Gamma_{qq}^p - \partial_q \Gamma_{pq}^p + \Gamma_{kp}^p \Gamma_{qq}^k - \Gamma_{qk}^p \Gamma_{pq}^k \quad (9)$$

$$= 0 \quad (10)$$

since all terms involve components of the form Γ_{ij}^p . Therefore, the Ricci tensor is also zero: $R_{ij} = 0$ as is the curvature scalar $R = 0$. The space is flat.