FALLING INTO A BLACK HOLE: TIDAL FORCES

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 19; Problem P19.9.

Using the Riemann tensor, we can get an idea of the force felt by someone falling into a black hole. Recall the original definition of the Riemann tensor was in terms of the equation of geodesic deviation:

$$\ddot{\mathbf{n}}^i = -R^i{}_{j\ell m}u^m u^j n^\ell \tag{1}$$

where $\mathbf{n}$ is the four-vector separating two infinitesimally close geodesics, and u is the four-velocity of an object in freefall along one of the geodesics.

For an object, such as a person, that has a large enough size that different parts of the object would, if they weren't connected, follow different geodesics, a tension force is felt as the various geodesics that pass through different parts of the object diverge during the object's journey. Suppose our unfortunate person is falling feet first into a black hole (we'll assume that the person started at rest very far away from the black hole). If we set up a locally inertial frame (LIF) at the person's centre of mass and align the person's local z axis with the radial direction in the Schwarzschild (S) metric, then we've seen that we can write the geodesic deviation as

$$\frac{d^2n^i}{dt^2} = -R^i{}_{t\ell t}n^\ell \tag{2}$$

In the case of the falling person, we can look at the z direction, since this is where most of the tidal effects will be felt. In that case, we get, using the person's LIF as the reference frame:

$$\ddot{n}^z = -R^z{}_{t\ell t}n^\ell \tag{3}$$

If we neglect separations in the x and y directions, this becomes

$$\ddot{n}^z = -R^z{}_{tzt}n^z \tag{4}$$

To get the Riemann component, we can use the symmetry of the tensor:

$$g_{za}R^{a}_{\ tzt} = R_{ztzt} = R_{tztz} = g_{ta}R^{a}_{\ ztz} \tag{5}$$

In the LIF, $g_{ij} = \eta_{ij}$ so this equation becomes

$$R^{z}_{\ tzt} = -R^{t}_{\ ztz} \tag{6}$$

and we worked out the RHS in the last post, so we have

$$R^{z}_{\ tzt} = -\frac{2GM}{r^3} \tag{7}$$

The acceleration of the z separation of the two geodesics is then

$$\ddot{n}^z = \frac{2GM}{r^3} n^z \tag{8}$$

which we can rewrite with r as a function of the acceleration felt at that distance from the black hole:

$$r = \left[\frac{2GMn^z}{\ddot{n}^z}\right]^{1/3} \tag{9}$$

To see how long it takes the person to fall from this distance to $r = 0$, we can use the formula we derived earlier:

$$\Delta\tau = \frac{\pi r^{3/2}}{\sqrt{8GM}} \tag{10}$$

So the time measured by the observer is

$$\Delta\tau = \frac{\pi}{2}\sqrt{\frac{n^z}{\ddot{n}^z}} \tag{11}$$

To put this in practical terms, a typical person can handle up to about 5g (five times the acceleration due to gravity at the Earth's surface) along their vertical direction before losing consciousness. If we take $n^z \approx 1$ m (about half a person's height) and $\ddot{n}^z \approx 50 \text{ m s}^{-2}$ then the time from first experiencing this force to annihilation at the singularity at the centre of the black hole is about

$$\Delta\tau \approx 0.2 \text{ sec} \tag{12}$$

Using Moore's estimate of the speed of pain impulses (around 1 m/sec) any pain resulting from this probably wouldn't be felt before the person gets annihilated.