ENERGY (NOT MASS) IS THE SOURCE OF GRAVITY

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 20; Box 20.1.

When we examined the tidal effect, first in Newtonian physics and then in general relativity we found that the Newtonian tidal deviation equation is

$$\frac{d^2n^i}{dt^2} = -\eta^{ij}n^k\frac{\partial^2\Phi}{\partial x^k\partial x^j} \tag{1}$$

where Φ is the gravitational potential function, and n^i is the deviation vector. In general relativity, the analogous result is the equation of geodesic deviation:

$$\ddot{\mathbf{n}}^i = -R^i_{\;j\ell m}u^m u^j n^\ell \tag{2}$$

which involves the Riemann tensor.

In classical physics, there is a strong parallel between gravitational and electromagnetic forces. Both are inverse-square and conservative forces, so the electric field due to a static collection of charges and the gravitational field due to a static collection of masses can each be derived as the gradient of a potential. As a result, Gauss's law applies to both types of field, so we can write

$$\nabla\cdot\mathbf{E} = \frac{\rho_E}{\epsilon_0} \tag{3}$$

$$\nabla\cdot\mathbf{g} = -4\pi G\rho_g \tag{4}$$

where $\mathbf{E}$ is the electric field, $\mathbf{g}$ is the gravitational field, ρ_E is the charge density and ρ_g is the mass density. The difference in sign is because the force between like charges is repulsive, while for gravity all forces are attractive.

If we write the field as the negative gradient of a potential, we get for gravity:

$$\mathbf{g} = -\nabla\Phi \tag{5}$$

$$\nabla^2\Phi = 4\pi G\rho_g \tag{6}$$

Comparing 1 and 2, we see that the Riemann tensor plays a similar role to $\partial_k \partial_j \Phi$ in Newtonian theory, so we might expect that the relativistic form of 6 will involve the Riemann tensor.

First, we look at translating the role of the mass density ρ_g from Newtonian to relativistic physics. In relativity, there is an argument that shows that we need to consider energy density as the source of gravity, rather than simply mass density. The argument goes like this. Suppose we take mass as the source of gravity, and we have a collection of matter and anti-matter localized near some point in space. Since both matter and anti-matter possess "normal" mass (there is no such thing as anti-mass), an object that starts off at rest a long distance from this mass will accelerate towards it, gaining kinetic energy as it moves faster. Suppose that this object passes through the matter/anti-matter particles without hitting any of them. If the matter and anti-matter remained in the same form, it would now exert a gravitational force on the receding object, causing it to slow down as it moved away, eventually coming to rest a long way from the mass. However, if the matter and anti-matter now collide with and annihilate each other, their mass is converted into photons which have no rest mass. If the source of gravity were mass (in the form of rest mass), then after annihilation, the photons would no longer exert any gravitational force, so the receding object would not slow down. It would reach infinity with the same kinetic energy it had when it passed through the matter/anti-matter. The idea is that this violates the law of conservation of energy.

If we consider *energy* rather than mass to be the source of gravity, then the photons will exert the same gravitational force on the receding object as the original matter/anti-matter, so the object will slow down as it recedes and eventually stop.

This argument seems logical, but there is one point that bothers me. In classical physics, an object falling towards a mass is said to have a combination of potential and kinetic energy, and it is this sum that remains constant. Thus when the object is far from the mass and at rest, all its energy is potential, and as the object falls and gains speed, the potential energy gets converted to kinetic energy. In the case of an object falling towards the matter/anti-matter, this is still true. However, if the matter and anti-matter annihilate each other as the particle is receding, then if we consider mass as the source of gravity, the gravitational field vanishes, and so too does the potential energy. Thus the object's energy remains as kinetic energy as it recedes, which would seem to satisfy the law of conservation of energy.

Perhaps a better way to look at it is like this. Suppose the matter/anti-matter annihilates when the object has receded to a point where it has slowed down a bit. Then the potential energy vanishes and the object retains part of the original kinetic energy. In this case, there is a net loss of energy since the potential energy just disappears off the scene. There is no loss of energy in the matter/anti-matter system itself, since the original rest mass gets converted into an equivalent amount of energy when the particles produce photons. This paradox is resolved if we take energy rather than mass as the source of gravity.

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