

## STRESS-ENERGY TENSOR AT THE CENTRE OF THE SUN

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 20; Problem 20.1.

As a numerical example of the stress-energy tensor, we can look at the hydrogen ion plasma at the centre of the sun. If we regard this plasma as a perfect fluid (implying no interaction between particles, which isn't quite true since the particles in question are protons and electrons, so there is definitely an electrical interaction, but at such high temperatures, this probably isn't all that important), and consider the rest frame of the fluid, then

$$(1) \quad T^{ij} = \begin{bmatrix} \rho_0 & 0 & 0 & 0 \\ 0 & P_0 & 0 & 0 \\ 0 & 0 & P_0 & 0 \\ 0 & 0 & 0 & P_0 \end{bmatrix}$$

where  $\rho_0$  is the energy density of the fluid and  $P_0$  is the pressure, both measured in the fluid's rest frame. To work out these two numbers, we need a bit of data.

It is believed that the density of the plasma is  $160 \text{ g cm}^{-3} = 1.6 \times 10^8 \text{ g m}^{-3}$  and the temperature is around  $1.5 \times 10^7 \text{ K}$ . Since the atomic weight of hydrogen is about 1 gram, and there are Avogadro's number ( $6.02 \times 10^{23}$ ) of atoms in one atomic weight, we can work out how many atoms  $N_a$  there are in 1 cubic metre:

$$(2) \quad N_a = (1.6 \times 10^8) (6.02 \times 10^{23}) = 9.632 \times 10^{31}$$

Since all the atoms are ionized, the actual number of particles is twice this, since we have one proton and one electron per atom. Thus

$$(3) \quad N = 2N_a = 1.926 \times 10^{32}$$

Now we can use the ideal gas law in the form  $P_0 V = N k_B T$ , where  $k_B$  is Boltzmann's constant which in GR units is

$$\begin{aligned}
 (4) \quad k_B &= 1.3806 \times 10^{-23} \text{ m}^2 \text{ kg K}^{-1} \text{ s}^{-2} \\
 (5) \quad &= \frac{1.3806 \times 10^{-23}}{c^2} \text{ kg K}^{-1} \\
 (6) \quad &= 1.536 \times 10^{-40} \text{ kg K}^{-1}
 \end{aligned}$$

(Note that the value given in Moore's book is incorrect.) Taking  $V = 1 \text{ m}^3$ , we get for the pressure:

$$\begin{aligned}
 (7) \quad P_0 &= (1.926 \times 10^{32}) (1.536 \times 10^{-40}) (1.5 \times 10^7) \\
 (8) \quad &= 0.4438 \text{ kg m}^{-3} \\
 (9) \quad &= 0.4438c^2 \text{ N m}^{-2} \\
 (10) \quad &= 3.99 \times 10^{16} \text{ Pascals}
 \end{aligned}$$

(For comparison, the atmospheric pressure at sea level on the Earth is around  $10^5$  Pascals.)

To work out the energy density, the kinetic energy of a particle in an ideal gas is, from statistical mechanics,  $\frac{3}{2}k_B T$  and its rest mass will be either the mass of the proton  $m_p$  or the mass of the electron  $m_e$ , so the total energy in 1 cubic metre is

$$\begin{aligned}
 (11) \quad \rho_0 &= \frac{3}{2}Nk_B T + \frac{1}{2}N(m_p + m_e) \\
 (12) \quad &= (0.666 + 1.61 \times 10^5) \text{ kg m}^{-3} \\
 (13) \quad &\approx 1.61 \times 10^5 \text{ kg m}^{-3}
 \end{aligned}$$

Since the rest mass contribution to  $\rho_0$  is much greater than the kinetic energy, we're justified in assuming the system is non-relativistic in speed. Also by comparing with 8 we find that  $\rho_0 \gg P_0$ . Even at the centre of a star, the overwhelming contribution to the stress-energy tensor comes from rest mass.

#### PINGBACKS

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