STRESS-ENERGY TENSOR AT THE CENTRE OF THE SUN

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 20; Problem 20.1.

As a numerical example of the stress-energy tensor, we can look at the hydrogen ion plasma at the centre of the sun. If we regard this plasma as a perfect fluid (implying no interaction between particles, which isn't quite true since the particles in question are protons and electrons, so there is definitely an electrical interaction, but at such high temperatures, this probably isn't all that important), and consider the rest frame of the fluid, then

$$T^{ij}=\left[\begin{array}{cccc}\rho_{0} & 0 & 0 & 0\\0 & P_{0} & 0 & 0\\0 & 0 & P_{0} & 0\\0 & 0 & 0 & P_{0}\end{array}\right] \tag{1}$$

where ρ_0 is the energy density of the fluid and P_0 is the pressure, both measured in the fluid's rest frame. To work out these two numbers, we need a bit of data.

It is believed that the density of the plasma is 160 g cm$^{-3}=1.6\times10^{8}$ g m^{-3} and the temperature is around 1.5×10^{7} K. Since the atomic weight of hydrogen is about 1 gram, and there are Avogadro's number (6.02×10^{23}) of atoms in one atomic weight, we can work out how many atoms N_a there are in 1 cubic metre:

$$N_{a}=\left(1.6\times10^{8}\right)\left(6.02\times10^{23}\right)=9.632\times10^{31} \tag{2}$$

Since all the atoms are ionized, the actual number of particles is twice this, since we have one proton and one electron per atom. Thus

$$N=2N_{a}=1.926\times10^{32} \tag{3}$$

Now we can use the ideal gas law in the form $P_0V=Nk_BT$, where k_B is Boltzmann's constant which in GR units is

$$k_B = 1.3806\times10^{-23}\text{ m}^2\text{ kg K}^{-1}\text{s}^{-2} \tag{4}$$
$$= \frac{1.3806\times10^{-23}}{c^2}\text{ kg K}^{-1} \tag{5}$$
$$= 1.536\times10^{-40}\text{ kg K}^{-1} \tag{6}$$

(Note that the value given in Moore's book is incorrect.) Taking $V = 1\text{ m}^3$, we get for the pressure:

$$P_0 = \left(1.926\times10^{32}\right)\left(1.536\times10^{-40}\right)\left(1.5\times10^{7}\right) \tag{7}$$
$$= 0.4438\text{ kg m}^{-3} \tag{8}$$
$$= 0.4438c^2\text{ N m}^{-2} \tag{9}$$
$$= 3.99\times10^{16}\text{ Pascals} \tag{10}$$

(For comparison, the atmospheric pressure at sea level on the Earth is around 10^5 Pascals.)

To work out the energy density, the kinetic energy of a particle in an ideal gas is, from statistical mechanics, $\frac{3}{2}k_BT$ and its rest mass will be either the mass of the proton m_p or the mass of the electron m_e, so the total energy in 1 cubic metre is

$$\rho_0 = \frac{3}{2}Nk_BT+\frac{1}{2}N\left(m_p+m_e\right) \tag{11}$$
$$= \left(0.666+1.61\times10^{5}\right)\text{ kg m}^{-3} \tag{12}$$
$$\approx 1.61\times10^{5}\text{ kg m}^{-3} \tag{13}$$

Since the rest mass contribution to ρ_0 is much greater than the kinetic energy, we're justified in assuming the system is non-relativistic in speed. Also by comparing with 8 we find that $\rho_0 \gg P_0$. Even at the centre of a star, the overwhelming contribution to the stress-energy tensor comes from rest mass.

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