STRESS-ENERGY TENSOR: RELATIVISTIC PERFECT FLUID

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 20; Problem 20.2.

We saw in the last post that even at the centre of the sun, the energy of the protons and electrons in the plasma is still non-relativistic and the pressure is still a very small fraction of the energy density. An interesting question is: how fast do particles need to move in a perfect fluid in order for the pressure to become significant in the stress-energy tensor?

Let's say we want the pressure to be 1% of the energy density, as measured in the fluid's rest frame. The energy density of a plasma containing N particles, each of mass m in a volume V is

$$\rho_0 = \gamma m \frac{N}{V} \tag{1}$$

If we take a simplified case where all particles are moving at the same speed v, but in random directions, then the four-momentum is

$$\mathbf{p} = \gamma m \left[1, v^x, v^y, v^z\right] \tag{2}$$

$$v = \sqrt{(v^x)^2 + (v^y)^2 + (v^z)^2} \tag{3}$$

When deriving the stress-energy tensor, we found that the pressure due to a particle moving with an x component of speed v^x could be written as

$$\Delta P = \frac{(p^x)^2}{p^t V} = \frac{(\gamma m v^x)^2}{\gamma m V} = \gamma m \frac{(v^x)^2}{V} \tag{4}$$

With N particles, the total pressure is (assuming that v^x averages out to the same value for all particles):

$$P = \gamma m N \frac{(v^x)^2}{V} \tag{5}$$

Further, we can assume that on average $v^x = v^y = v^z$ so we can write this as

$$P = \frac{\gamma m N}{V}\frac{v^2}{3} \tag{6}$$

If the pressure is to be 1% of the energy density, then

$$\frac{\gamma m N}{V}\frac{v^2}{3} = 0.01\gamma m\frac{N}{V} \tag{7}$$

$$v = \sqrt{0.03} = 0.173 \tag{8}$$

Thus the particles need to be moving at a significant fraction of the speed of light in order for the pressure to be significant.