METRIC TENSOR AS A STRESS-ENERGY TENSOR

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 20; Problem 20.7.

A curious possibility for the stress-energy tensor is

$$T^{ij}=-\Lambda g^{ij} \tag{1}$$

where Λ is a positive constant and g^{ij} is any metric tensor. In a local inertial frame (LIF), $g^{ij}=\eta^{ij}$ and

$$T^{ij}=\Lambda\left[\begin{array}{cccc}
1 & 0 & 0 & 0\\
0 & -1 & 0 & 0\\
0 & 0 & -1 & 0\\
0 & 0 & 0 & -1
\end{array}\right] \tag{2}$$

Note that this tensor satisfies the energy conservation equation $\nabla_{i}T^{ij}=0$ as the covariant derivative $\nabla_{i}g^{ij}$ is always zero.

Comparing this with the form of T^{ij} for a perfect fluid in its LIF

$$T^{ij}=\left[\begin{array}{cccc}
\rho & 0 & 0 & 0\\
0 & P & 0 & 0\\
0 & 0 & P & 0\\
0 & 0 & 0 & P
\end{array}\right] \tag{3}$$

we see that the energy density is $\rho=\Lambda>0$ and the pressure is $P=-\Lambda<0$. Such a tensor cannot arise from a perfect fluid because for such a fluid, for example

$$T^{xx}=\int\int\int N\left(p\right)\frac{\left(p^{x}\right)^{2}}{p^{t}V}dp^{x}dp^{y}dp^{z} \tag{4}$$

The integrand is instrinsically non-negative because $N\left(p\right)$ is the number of particles in volume V with a magnitude of momentum p and must be non-negative. The component $p^{t}=\gamma m$ for massive particles or $p^{t}=E$ for photons, but in either case it too is positive. The numerator $(p^x)^2$, being a square, is also non-negative. Thus for a perfect fluid in its LIF, $T^{xx} \geq 0$.

Although this tensor cannot be that of a perfect fluid, it does play a role in general relativity as we'll hopefully see a bit later.

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