

## METRIC TENSOR AS A STRESS-ENERGY TENSOR

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 20; Problem 20.7.

A curious possibility for the stress-energy tensor is

$$T^{ij} = -\Lambda g^{ij} \quad (1)$$

where  $\Lambda$  is a positive constant and  $g^{ij}$  is any metric tensor. In a local inertial frame (LIF),  $g^{ij} = \eta^{ij}$  and

$$T^{ij} = \Lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad (2)$$

Note that this tensor satisfies the energy conservation equation  $\nabla_i T^{ij} = 0$  as the covariant derivative  $\nabla_i g^{ij}$  is always zero.

Comparing this with the form of  $T^{ij}$  for a perfect fluid in its LIF

$$T^{ij} = \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{bmatrix} \quad (3)$$

we see that the energy density is  $\rho = \Lambda > 0$  and the pressure is  $P = -\Lambda < 0$ . Such a tensor cannot arise from a perfect fluid because for such a fluid, for example

$$T^{xx} = \int \int \int N(p) \frac{(p^x)^2}{p^t V} dp^x dp^y dp^z \quad (4)$$

The integrand is intrinsically non-negative because  $N(p)$  is the number of particles in volume  $V$  with a magnitude of momentum  $p$  and must be non-negative. The component  $p^t = \gamma m$  for massive particles or  $p^t = E$  for photons, but in either case it too is positive. The numerator  $(p^x)^2$ , being a square, is also non-negative. Thus for a perfect fluid in its LIF,  $T^{xx} \geq 0$ .

Although this tensor cannot be that of a perfect fluid, it does play a role in general relativity as we'll hopefully see a bit later.

#### PINGBACKS

Pingback: [Stress-energy tensor: negative pressure revisited](#)

Pingback: [Dominant energy condition](#)