# STRESS-ENERGY TENSOR: NEGATIVE PRESSURE REVISITED

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 20; Problem 20.9.

Suppose we have a stress-energy tensor of the form

$$T^{ij}=\left[\begin{array}{cccc}\rho & 0 & 0 & 0\\ 0 & \alpha\rho & 0 & 0\\ 0 & 0 & \alpha\rho & 0\\ 0 & 0 & 0 & \alpha\rho\end{array}\right] \tag{0.1}$$

where $\rho$ is the energy density of the perfect fluid in its rest frame and $\alpha$ is a scalar constant.

If an observer that moves with four-velocity $\mathbf{u}_{obs}$ relative to the fluid's frame is to see the energy density in his frame as positive, this imposes a constraint on the possible values of $\alpha$. We can analyze this by looking at the situation in the observer's local orthonormal frame (LOF). In this case, the global frame is the rest frame of the fluid. In the observer's own local frame, because his metric is flat and the observer is not moving relative to himself, $\mathbf{u}=[1,0,0,0]$. That is, in the local frame, $\mathbf{u}=\mathbf{o}_t$. Therefore, $\mathbf{u}_{obs}$ in the global frame is the transformed version of $\mathbf{o}_t$ so in the global frame

$$\mathbf{o}_t = \mathbf{u}_{obs} \tag{0.2}$$

$$= \left[\gamma,\gamma v^x,\gamma v^y,\gamma v^z\right] \tag{0.3}$$

We can find the energy density seen by the moving observer by applying the transformation for the stress-energy tensor. The transformation of the energy density is

$$T_{obs}^{tt}=(\rho+P)\gamma^2-P \tag{0.4}$$

with $P=\alpha\rho$, so we get

$$T_{obs}^{tt}=\gamma^2(1+\alpha)\rho-\alpha\rho>0 \tag{0.5}$$

Since the energy density $\rho$ in the rest frame is known to be positive, we can cancel it off and get

$$\alpha > -\frac{\gamma^2}{\gamma^2 - 1} \tag{0.6}$$

If we want this condition to be satisfied for *all* possible observers, then we must take the limit as the observer's speed $v \to 1$, or $\gamma \to \infty$, giving

$$\alpha > -1 \tag{0.7}$$

A negative $\alpha$ admits the possibility of a negative pressure, which we've already seen can't correspond to a perfect fluid, even though it gives an acceptable stress-energy tensor.