

EINSTEIN EQUATION FOR A PERFECT FLUID

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 21; Problem 21.3.

We wish to prove that $G^{ij} = 0$ if and only if $R^{ij} = 0$. The Einstein tensor is defined as

$$(0.1) \quad G^{ij} \equiv R^{ij} - \frac{1}{2}g^{ij}R$$

Clearly if the Ricci tensor $R^{ij} = 0$ then $G^{ij} = 0$ (since the curvature scalar is the contraction of the Ricci tensor: $R = g_{ij}R^{ij}$). To prove the converse, suppose $G^{ij} = 0$. Then we can multiply both sides by g_{ij} to get

$$(0.2) \quad 0 = g_{ij}R^{ij} - \frac{1}{2}g_{ij}g^{ij}R$$

$$(0.3) \quad = R - 2R$$

$$(0.4) \quad R = 0$$

since $g_{ij}g^{ij} = 4$. With $G^{ij} = 0$ and $R = 0$, 0.1 tells us that $R^{ij} = 0$. QED.