EINSTEIN EQUATION FOR A PERFECT FLUID

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 21; Problem 21.3.

We wish to prove that $G^{ij}=0$ if and only if $R^{ij}=0$. The Einstein tensor is defined as

$$G^{ij}\equiv R^{ij}-\frac{1}{2}g^{ij}R \tag{1}$$

Clearly if the Ricci tensor $R^{ij}=0$ then $G^{ij}=0$ (since the curvature scalar is the contraction of the Ricci tensor: $R=g_{ij}R^{ij}$). To prove the converse, suppose $G^{ij}=0$. Then we can multiply both sides by g_{ij} to get

$$\begin{aligned}
0 &= g_{ij}R^{ij}-\frac{1}{2}g_{ij}g^{ij}R && (2)\\
&= R-2R && (3)\\
R &= 0 && (4)
\end{aligned}$$

since $g_{ij}g^{ij}=4$. With $G^{ij}=0$ and $R=0$, 1 tells us that $R^{ij}=0$. QED.