

EINSTEIN EQUATION: ALTERNATIVE FORM

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 21; Box 21.3.

The general relativistic generalization of Newton's law of gravity is

$$(1) \quad G^{ij} + \Lambda g^{ij} = \kappa T^{ij}$$

where the Einstein tensor is defined in terms of the Ricci tensor and the curvature scalar as

$$(2) \quad G^{ij} \equiv R^{ij} - \frac{1}{2}g^{ij}R$$

We can write this in a different form that is sometimes easier to use in calculations. Eliminating G^{ij} we have

$$(3) \quad R^{ij} - \frac{1}{2}g^{ij}R + \Lambda g^{ij} = \kappa T^{ij}$$

Multiplying both sides by g_{ij} we get

$$(4) \quad g_{ij}R^{ij} - \frac{1}{2}g_{ij}g^{ij}R + \Lambda g_{ij}g^{ij} = \kappa g_{ij}T^{ij}$$

Because the tensor g_{ij} is the inverse of g^{ij} , their product gives the identity matrix of rank 4 (this can be seen by doing the calculation in a local inertial frame where $g_{ij} = \eta_{ij}$ and noting that since it's a tensor equation, it's valid in all coordinate systems). That is

$$(5) \quad g_{ij}g^{jk} = \delta_i^k$$

so if we contract the δ_j^k tensor we just sum up its diagonal elements and since these are all 1 (and there are four rows), we get

$$(6) \quad \delta_k^k = 4$$

Returning to 4 we get

$$(7) \quad g_{ij}R^{ij} - 2R + 4\Lambda = \kappa g_{ij}T^{ij}$$

Since the curvature scalar is given by

$$(8) \quad R \equiv g_{ij}R^{ij}$$

and the stress-energy scalar is

$$(9) \quad T \equiv g_{ij}T^{ij}$$

we get

$$(10) \quad -R + 4\Lambda = \kappa T$$

Multiplying this by $-\frac{1}{2}g^{ij}$ and subtracting from 3 we have

$$(11) \quad R^{ij} - \Lambda g^{ij} = \kappa \left(T^{ij} - \frac{1}{2}g^{ij}T \right)$$

Isolating the Ricci tensor gives us the alternative form of the Einstein equation:

$$(12) \quad \boxed{R^{ij} = \kappa \left(T^{ij} - \frac{1}{2}g^{ij}T \right) + \Lambda g^{ij}}$$

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