

EINSTEIN EQUATION FOR A PERFECT FLUID

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 21; Problem 21.3.

The Einstein equation can be written as

$$R^{ij} = \kappa \left(T^{ij} - \frac{1}{2} g^{ij} T \right) + \Lambda g^{ij} \quad (1)$$

For a perfect fluid the stress-energy tensor is

$$T^{ij} = (\rho_0 + P_0) u^i u^j + P_0 g^{ij} \quad (2)$$

where u^i is the four-velocity, ρ_0 is the energy density and P_0 is the pressure. We have for the stress-energy scalar:

$$T = (\rho_0 + P_0) g_{ij} u^i u^j + P_0 g_{ij} g^{ij} \quad (3)$$

$$= -(\rho_0 + P_0) + 4P_0 \quad (4)$$

$$= 3P_0 - \rho_0 \quad (5)$$

since $g_{ij} g^{ij} = 4$ and $g_{ij} u^i u^j = -1$ in any coordinate system.

The stress energy term in 1 is therefore

$$T^{ij} - \frac{1}{2} g^{ij} T = (\rho_0 + P_0) u^i u^j + P_0 g^{ij} - \frac{1}{2} g^{ij} (3P_0 - \rho_0) \quad (6)$$

$$= (\rho_0 + P_0) u^i u^j + \frac{1}{2} g^{ij} (\rho_0 - P_0) \quad (7)$$

In a local orthogonal frame (LOF) in which the fluid is at rest $u^t = 1$, $u^i = 0$ for $i = x, y, z$ and $g^{ij} = \eta^{ij}$, so

$$T^{tt} - \frac{1}{2}g^{tt}T = \rho_0 + P_0 - \frac{1}{2}(\rho_0 - P_0) \quad (8)$$

$$= \frac{1}{2}(\rho_0 + 3P_0) \quad (9)$$

$$T^{xx} - \frac{1}{2}g^{xx}T = \frac{1}{2}(\rho_0 - P_0) \quad (10)$$

$$T^{yy} - \frac{1}{2}g^{yy}T = \frac{1}{2}(\rho_0 - P_0) \quad (11)$$

$$T^{zz} - \frac{1}{2}g^{zz}T = \frac{1}{2}(\rho_0 - P_0) \quad (12)$$

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