

GRAVITY CAN'T EXIST IN 3 SPACETIME DIMENSIONS EITHER

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 21; Problem 21.6.

We've seen that the Einstein equation doesn't allow gravity to exist in 2 spacetime dimensions. Here we'll look at a demonstration that gravity also cannot exist in 3 spacetime dimensions of t , x and y .

Because of the symmetries of the Riemann tensor, there are six independent, (possibly) non-zero components in 3 dimensions. We can take these components to be R_{xtxt} , R_{ytyt} , R_{xtyt} , R_{txxy} , R_{txyt} and R_{tyxy} . As before, we use the Einstein equation

$$(1) \quad R^{ij} = \kappa \left(T^{ij} - \frac{1}{2} g^{ij} T \right) + \Lambda g^{ij}$$

with $\Lambda = 0$ and consider a vacuum so that $T^{ij} = T = 0$, meaning that $R^{ij} = 0$. We'll look at a local inertial frame (LIF), where the metric is $g^{ij} = \eta^{ij}$. Then we get

$$(2) \quad R_{ij} = R^a_{\ iaj}$$

$$(3) \quad = \eta^{ak} R_{kiaj}$$

$$(4) \quad = -R_{tijt} + R_{xixj} + R_{yiyj}$$

Now we look at the 6 independent components of R_{ij} . Because $R_{ijkm} = -R_{jikm} = -R_{ijmk}$, any component with either the first two indices or last two indices equal is zero, so we get

$$(5) \quad R_{tt} = R_{xtxt} + R_{ytyt} = 0$$

$$(6) \quad R_{xx} = -R_{ttxx} + R_{yxyx} = 0$$

$$(7) \quad R_{yy} = -R_{tyty} + R_{xyxy} = 0$$

$$(8) \quad R_{tx} = R_{ytxx} = 0$$

$$(9) \quad R_{ty} = R_{xtxy} = 0$$

$$(10) \quad R_{xy} = -R_{txty} = 0$$

The last 3 equations show that 3 of the Riemann components are zero. The first 3 equations can be rewritten using the symmetries of the Riemann tensor:

$$(11) \quad R_{tt} = R_{xtxt} + R_{ytyt} = 0$$

$$(12) \quad R_{xx} = -R_{xtxt} + R_{xyxy} = 0$$

$$(13) \quad R_{yy} = -R_{ytyt} + R_{xyxy} = 0$$

Solving these equations gives

$$(14) \quad R_{xtxt} = R_{ytyt} = R_{xyxy} = 0$$

Thus all 6 components of the Riemann tensor are zero, showing that 3d spacetime must be flat and gravity cannot exist in 3 spacetime dimensions. (As usual, a tensor equation valid in a LIF is valid in all coordinate systems, so the conclusion is general.)