

GRAVITOELECTRIC AND GRAVITOMAGNETIC DENSITIES FOR THE VACUUM

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 22, Problem 22.2.

The gravitoelectric, gravitomagnetic and curvature densities are

$$\rho_g = 2T_{tt} - \eta_{tt}T \quad (1)$$

$$\Pi_i = -T_{ti} + \frac{1}{2}\eta_{ti}T \quad (2)$$

$$\rho_c = 2T_{ii} - \eta_{ii}T \quad (3)$$

where i is a spatial index.

The vacuum stress-energy tensor is given in terms of the metric and cosmological constant as

$$T_{ij,vac} = -g_{ij} \frac{\Lambda}{8\pi G} \quad (4)$$

Since $\frac{\Lambda}{8\pi G}$ is very small and $g_{ij} = \eta_{ij} + h_{ij}$ for a perturbation h_{ij} , we can approximate:

$$T_{ij,vac} \approx -\eta_{ij} \frac{\Lambda}{8\pi G} \quad (5)$$

$$T = -\eta_{ij}\eta^{ij} \frac{\Lambda}{8\pi G} \quad (6)$$

$$= -\frac{\Lambda}{2\pi G} \quad (7)$$

Therefore

$$\rho_g = -\frac{\Lambda}{\pi G} \eta_{tt} \left(-\frac{1}{4} + \frac{1}{2} \right) \quad (8)$$

$$= -\frac{\Lambda}{4\pi G} \quad (9)$$

$$\Pi_i = \eta_{ti} \frac{\Lambda}{8\pi G} + \eta_{ti} T \quad (10)$$

$$= 0 \quad (11)$$

$$\rho_c = -\frac{\Lambda}{\pi G} \eta_{ii} \left(\frac{1}{4} - \frac{1}{2} \right) \quad (12)$$

$$= \frac{\Lambda}{4\pi G} \quad (13)$$

In the third line $\eta_{ti} = 0$ if i is a spatial index, and in the fifth line $\eta_{ii} = +1$.