GRAVITOELECTRIC AND GRAVITOMAGNETIC DENSITIES FOR THE VACUUM

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 22, Problem 22.2.

The gravitoelectric, gravitomagnetic and curvature densities are

$$\rho_g = 2T_{tt} - \eta_{tt}T \tag{1}$$

$$\Pi_i = -T_{ti} + \frac{1}{2}\eta_{ti}T \tag{2}$$

$$\rho_c = 2T_{ii} - \eta_{ii}T \tag{3}$$

where i is a spatial index.

The vacuum stress-energy tensor is given in terms of the metric and cosmological constant as

$$T_{ij,vac} = -g_{ij}\frac{\Lambda}{8\pi G} \tag{4}$$

Since $\frac{\Lambda}{8\pi G}$ is very small and $g_{ij} = \eta_{ij} + h_{ij}$ for a perturbation h_{ij}, we can approximate:

$$T_{ij,vac} \approx -\eta_{ij}\frac{\Lambda}{8\pi G} \tag{5}$$

$$T = -\eta_{ij}\eta^{ij}\frac{\Lambda}{8\pi G} \tag{6}$$

$$= -\frac{\Lambda}{2\pi G} \tag{7}$$

Therefore

$$\rho_g = -\frac{\Lambda}{\pi G}\eta_{tt}\left(-\frac{1}{4}+\frac{1}{2}\right) \tag{8}$$

$$= -\frac{\Lambda}{4\pi G} \tag{9}$$

$$\Pi_i = \eta_{ti}\frac{\Lambda}{8\pi G}+\eta_{ti}T \tag{10}$$

$$= 0 \tag{11}$$

$$\rho_c = -\frac{\Lambda}{\pi G}\eta_{ii}\left(\frac{1}{4}-\frac{1}{2}\right) \tag{12}$$

$$= \frac{\Lambda}{4\pi G} \tag{13}$$

In the third line $\eta_{ti} = 0$ if i is a spatial index, and in the fifth line $\eta_{ii} = +1$.