TIME DILATION

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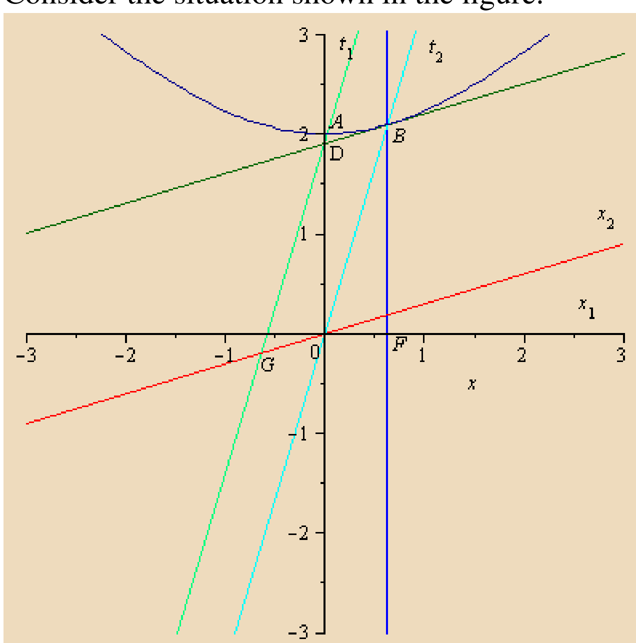
We saw in an earlier post that the events with a particular interval Δs^2 between them lie on an invariant hyperbola, and that this hyperbola can be used to calibrate the time or space intervals on the coordinate axes of two observers. We noted in passing that one effect of the postulates of relativity is *time dilation*, in which one observer believes the other observer's clocks run slow relative to himself.

We'll review the effect here and then resolve a common so-called paradox that many people believes invalidates relativity.

Consider the situation shown in the figure.



Observer O_2 moves with a speed v in the x direction relative to observer O_1 as usual. The coordinate systems of the two observers are shown on the diagram, and the dark blue hyperbola (with equation $x_1^2 - t_1^2 = -4$) passing through the points A and B defines all events with an interval $\Delta s^2 = -4$. This means that the point A, where the hyperbola intersects the t_1 axis, is the point where O_1's clock with world line $0A$ reads $t_1 = 2$. Observer O_2's clock whose world line is $0B$ reads $t_2 = 2$ at point B.

Since O_1 measures all events on a horizontal line passing through A as being simultaneous, and since B is not on this line, he will say that A and B occur at different times. As we saw in the earlier post, we can work out the coordinates of event B (by solving for the intersection of the hyperbola and the line $0B$), and they turn out to be

$$x_1 = \frac{2v}{\sqrt{1-v^2}} \tag{1}$$

$$t_1 = \frac{2}{\sqrt{1-v^2}} \tag{2}$$

That is, O_1 measures the time of event B as $t_1 = t_2/\sqrt{1-v^2}$ while O_2 measures it as $t_2 = 2$. Since $t_1 > t_2$, O_1 believes that more time has elapsed than O_2 does, so he believes that O_2's clock runs slow. This is the time dilation effect.

A paradox? The paradox (or so it is sometimes believed) is this: if O_1 believes O_2's clock runs slow, then surely O_2 must believe that O_1's clock runs fast. However, the principle of relativity (all inertial frames are equivalent) says that each observer should believe the *same* thing about the other, so that O_2 should measure O_1's clock as slow, not fast, relative to himself.

The reason why this apparent paradox occurs is that the measurement process which gives rise to the time dilation effect is not symmetric between the two observers. To see this, we must consider carefully what it is we are actually measuring.

The disparity occurs because we are comparing what the two observers are measuring as the time of event B. O_2 does this with one clock: the clock with world line $0B$. O_1, however, uses two clocks to do the measurement. The first clock is one whose world line is $0A$. This clock coincides with O_2's clock at the origin, where both observers agree that the time of this event is $t_1 = t_2 = 0$. However, after this event, the two clocks diverge and follow different world lines, so when the observers want to measure the time of event B, O_1 can't use the same clock that was used to measure the time at the origin, since that clock's world line doesn't go through event B. Instead he has to use the clock with world line FB. This looks fine to O_1 since he measures events 0 and F as simultaneous (they both lie on his x_1 axis so they both occur at $t_1 = 0$). However, O_2 disagrees that events 0 and F are simultaneous. According to O_2, the events that are simultaneous with event 0 are those that lie on the x_2 axis (the red line in the diagram), and it is clear that event F occurs before any event on this axis. Thus to O_2, O_1's clock ticks off *more* time in travelling from F to B than O_2's clock does in going from 0 to B. Thus both O_1 and O_2 will agree, after doing this analysis, that O_1's clock should read a later time than O_2's clock at event B.

Now let's look at event B from another viewpoint. According to O_2, events that are simultaneous with B must lie on the line passing through B and parallel to the x_2 axis; this line is shown in dark green in the figure. This line intersects the t_1 axis at event D, so O_2 measures the time of event D as $t_2 = 2$. What time does O_1 assign to this event? We can work this out by finding the coordinates of D. The equation of the dark green line which has slope v (as we saw in an earlier post), using the point-slope form of a straight line, is (we'll work it out for a general value of t_2 since we're trying to show the time dilation effect is symmetric):

$$t_1 - \frac{t_2}{\sqrt{1-v^2}} = v\left(x_1 - \frac{t_2 v}{\sqrt{1-v^2}}\right) \tag{3}$$

where we used the coordinates of B given above as the point through which this line passes. This line crosses the t_1 axis at $x_1 = 0$, so the time of event D as measured by O_1 is thus

$$\begin{aligned} t_1 &= \frac{t_2}{\sqrt{1-v^2}} - \frac{t_2 v^2}{\sqrt{1-v^2}} & (4)\\ &= t_2 \frac{1-v^2}{\sqrt{1-v^2}} & (5)\\ &= t_2\sqrt{1-v^2} & (6)\\ t_2 &= \frac{t_1}{\sqrt{1-v^2}} & (7) \end{aligned}$$

That is, O_2 now sees O_1's clock as running slow compared to his own. So the time dilation effect really does work both ways, and each observer really does see the other's clocks as running slow. In this last analysis, note that we are using *one* of O_1's clocks (the one with world line $0D$) and *two* of O_2's clocks (one with world line $0B$ to measure the time at the origin, and the other with world line GD (light green) to measure the time of event D). In this case, O_2 measures events 0 and G as simultaneous, but O_1 thinks G occurred before 0, so he thinks O_2 is measuring a longer time than O_1.

The key in understanding time dilation, and resolving the paradox, is to understand that the measurements involved in analyzing the times determined for a given event are *not* symmetric between the two observers: one observer always has to use one clock, and the other observer always has to use two clocks. This is because the two observers are moving relative to each other, so when one of O_1's clocks coincides with one of O_2's clocks, those two clocks can never be at the same place again, so one of the observers has to use a different clock to compare times between the two frames. The time dilation effect most definitely does *not* imply that *all* clocks in one frame run at a different rate from *all* clocks in another frame. You must be very careful about which clocks are being compared, and at which events the comparison takes place.

One final note: nowhere in this discussion of time have we made any assumption about the nature of the clocks being used to make the measurements. They could be mechanical clocks or atomic clocks or whatever. These effects arise entirely from the postulates of relativity, and the difference between relativity and Galilean physics is due entirely to the assumption of the constancy of the speed of light. The time dilation effect is real and is a property of time itself, not of the clocks used to measure it.

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