

## ANTI-HERMITIAN OPERATORS

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A hermitian operator is equal to its hermitian conjugate (which, remember, is the complex conjugate of the transpose of the matrix representing the operator). That is,

$$\hat{Q}^\dagger = \hat{Q} \quad (1)$$

This has the consequence that for inner products

$$\langle f|\hat{Q}g\rangle = \langle \hat{Q}^\dagger f|g\rangle \quad (2)$$

$$= \langle \hat{Q}f|g\rangle \quad (3)$$

An *anti-hermitian* operator is equal to the negative of its hermitian conjugate, that is

$$\hat{Q}^\dagger = -\hat{Q} \quad (4)$$

In inner products, this means

$$\langle f|\hat{Q}g\rangle = \langle \hat{Q}^\dagger f|g\rangle \quad (5)$$

$$= -\langle \hat{Q}f|g\rangle \quad (6)$$

The expectation value of an anti-hermitian operator is:

$$\langle f|\hat{Q}f\rangle = \langle \hat{Q}^\dagger f|f\rangle \quad (7)$$

$$= -\langle \hat{Q}f|f\rangle \quad (8)$$

$$= -\langle Q\rangle^* \quad (9)$$

But  $\langle f|\hat{Q}f\rangle = \langle Q\rangle$  so  $\langle Q\rangle = -\langle Q\rangle^*$ , which means the expectation value must be pure imaginary.

For two hermitian operators  $\hat{Q}$  and  $\hat{R}$  we have

$$[\hat{Q}, \hat{R}] = \hat{Q}\hat{R} - \hat{R}\hat{Q} \quad (10)$$

$$[\hat{Q}, \hat{R}]^\dagger = \hat{R}^\dagger \hat{Q}^\dagger - \hat{Q}^\dagger \hat{R}^\dagger \quad (11)$$

$$= \hat{R}\hat{Q} - \hat{Q}\hat{R} \quad (12)$$

$$= [\hat{R}, \hat{Q}] \quad (13)$$

$$= -[\hat{Q}, \hat{R}] \quad (14)$$

where we have used the hermitian property  $\hat{Q}^\dagger = \hat{Q}$  to get the third line. Thus the commutator of two hermitian operators is anti-hermitian.

If two operators  $\hat{S}$  and  $\hat{T}$  are anti-hermitian, a similar derivation shows that  $[\hat{S}, \hat{T}]^\dagger = -[\hat{S}, \hat{T}]$  also:

$$[\hat{S}, \hat{T}] = \hat{S}\hat{T} - \hat{T}\hat{S} \quad (15)$$

$$[\hat{S}, \hat{T}]^\dagger = \hat{T}^\dagger \hat{S}^\dagger - \hat{S}^\dagger \hat{T}^\dagger \quad (16)$$

$$= (-\hat{T})(-\hat{S}) - (-\hat{S})(-\hat{T}) \quad (17)$$

$$= -[\hat{S}, \hat{T}] \quad (18)$$

#### PINGBACKS

Pingback: Energy-time uncertainty relation