

BORN APPROXIMATION FOR A SPHERICAL DELTA FUNCTION SHELL

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Earlier we looked at scattering from a delta function spherical shell for a low energy incident particle, using partial wave analysis. This was a fairly complex task, as it involved matching interior and exterior wave functions at the delta function boundary.

Here, we'll calculate the scattering amplitude using the first Born approximation. For a spherically symmetric potential, the approximation is

$$f(\theta) \approx -\frac{2m}{\hbar^2 \kappa} \int_0^\infty V(r) r \sin(\kappa r) dr \quad (1)$$

where

$$\kappa \equiv 2k \sin \frac{\theta}{2} \quad (2)$$

For a delta function potential

$$V(r) = \alpha \delta(r - a) \quad (3)$$

where α is a constant representing the strength of the delta function, so we get

$$f(\theta) \approx -\frac{2m\alpha a}{\hbar^2 \kappa} \sin(\kappa a) \quad (4)$$

For low energy, $ka \ll 1$ so $\kappa a \ll 1$ as well, so $\sin(\kappa a) \approx \kappa a$, and we get

$$f(\theta) \approx -\frac{2m\alpha a^2}{\hbar^2} \quad (5)$$

Our earlier result using partial wave analysis is

$$f(\theta) \approx -\frac{\beta a}{1 + \beta} \quad (6)$$

where

$$\beta \equiv \frac{2m\alpha a}{\hbar^2} \quad (7)$$

This gives a differential cross section and total cross section of

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 = \beta^2 a^2 \quad (8)$$

$$\sigma = 4\pi\beta^2 a^2 \quad (9)$$

The low energy result 5 from the Born approximation is, in terms of β :

$$f(\theta) \approx -\beta a \quad (10)$$

so it agrees with the partial wave result if $\beta \ll 1$. This is equivalent to the condition that $\alpha \ll \hbar^2/2ma$, in other words, that the potential is weak. This was the main assumption in deriving the Born approximation, so in this limit, the results are consistent.