ELECTRON AS A CLASSICAL SPINNING SPHERE

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We can get a value for the rotation speed of an electron if we take it to be a classical spinning sphere of radius r. The spin angular momentum of the electron is

$$L = \frac{\hbar}{2} \tag{1}$$

The classical electron radius is taken to be

$$r = \frac{e^2}{4\pi\epsilon_0 mc^2} \tag{2}$$

Angular momentum can be written as $L = I\omega$, where I is the moment of inertia and ω is the angular velocity. For a solid sphere of mass m and radius r, $I = 2mr^2/5$ (see below). The equatorial speed on the sphere is $v = r\omega$, so using 1 and 2 we have

$$L = \frac{\hbar}{2} \tag{3}$$

$$= \frac{2}{5}mr^2\omega \tag{4}$$

$$\omega = \frac{5}{4}\frac{\hbar}{mr^2} \tag{5}$$

$$v = r\omega \tag{6}$$

$$= \frac{5}{4}\frac{\hbar}{mr} \tag{7}$$

$$= \frac{5}{4}\frac{4\pi\epsilon_0 c^2\hbar}{e^2} \tag{8}$$

In SI units, $\hbar = 1.0546\times 10^{-34} m^2 kg/s$, $1/4\pi\epsilon_0 = 8.9876\times 10^9\, N\, m^2/C^2$, $c = 3\times 10^8 m/s$, $e = 1.6\times 10^{-19} C$. Plugging in the numbers gives a speed of 5.16×10^{10} m s^{-1}, which is of course larger than the speed of light, so the classical analysis is inconsistent with relativity.

As an extra, here is a derivation of the moment of inertia of a sphere using cylindrical shells, which isn't the usual derivation. Suppose we divide the

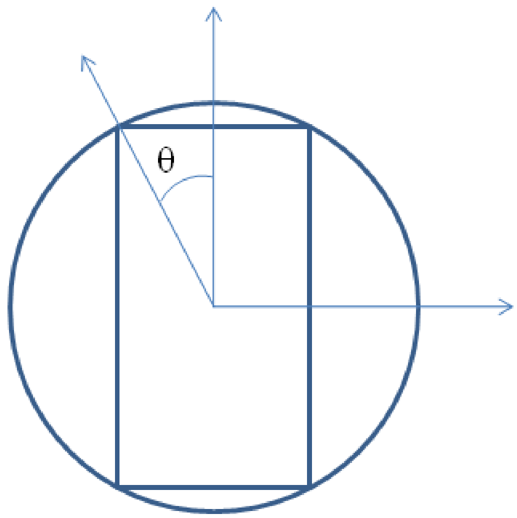

FIGURE 1. Cylindrical shell in a sphere.

sphere into a number of concentric cylindrical shells, whose axes coincide with the axis of rotation of the sphere (see Fig. 1 for side view). All points in the cylindrical shell are the same distance $r\sin\theta$ from the axis of the sphere. Since the radius of the cylinder is $r\sin\theta$ and its height is $2r\cos\theta$ the volume of the shell is $(2\pi r\sin\theta)(2r\cos\theta)dx$ where dx is the thickness of the shell. To get the thickness of the shell, consider Fig. 2.

On the right is a magnified view of the top of the shell. If we consider an increment in the angle θ, this generates an infinitesimal distance $r\cdot d\theta$ along the sphere. We want the horizontal increment corresponding to this distance. The angle between the horizontal and the tangent to the sphere at this point is θ (since these two lines are perpendicular to the two lines that generated θ in the diagram on the left), so the horizontal increment is $dx = r\cos\theta d\theta$. The volume of the shell is therefore

$$dV = 4\pi r^3 \sin\theta \cos^2\theta d\theta \tag{9}$$

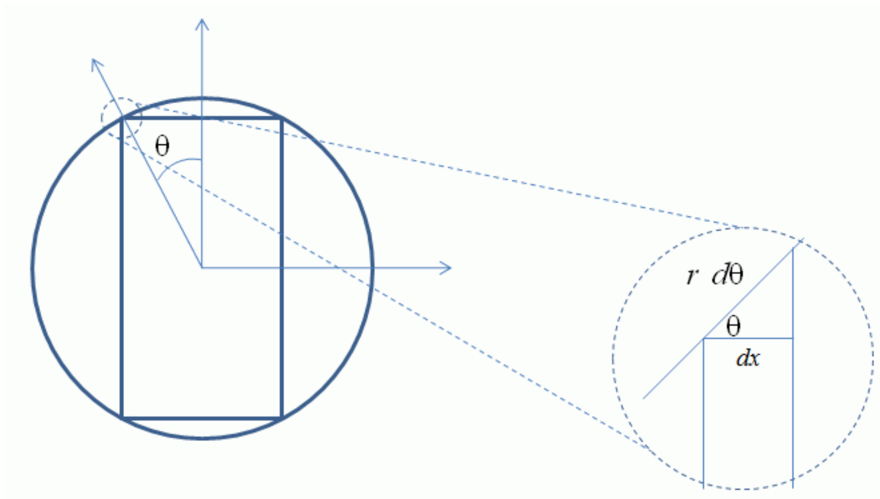

FIGURE 2. Thickness of cylindrical shell.

The mass of the shell is the density ρ multiplied by the volume, or ρdV. The moment of inertia of the shell is

$$dI = dV(r\sin\theta)^2 \tag{10}$$
$$= 4\pi r^5\rho\sin^3\theta\cos^2\theta d\theta \tag{11}$$
$$= 4\pi r^5\rho\sin\theta(\cos^2\theta - \cos^4\theta)d\theta \tag{12}$$

In this last form, the integral is easy, so we get

$$I = 4\pi r^5\rho\int_0^{\pi/2}\sin\theta(\cos^2\theta - \cos^4\theta)d\theta \tag{13}$$
$$= \frac{8}{15}\pi r^5\rho \tag{14}$$

The mass of the sphere is $m = 4\pi r^3\rho/3$, so this formula becomes

$$I = \frac{2}{5}mr^2 \tag{15}$$

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