EVEN AND ODD SOLUTIONS TO THE SCHRÖDINGER EQUATION

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The time-independent Schrödinger equation can be solved by separation of variables, with the spatial part satisfying

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+V(x)\psi=E\psi \tag{1}$$

with $V(x)$ being the potential function and E being one of the allowable energies.

If the potential is even, so that $V(x)=V(-x)$, then $\psi(x)$ can be taken as even or odd. This follows by considering the Schrödinger equation with x replaced by $-x$:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(-x)}{dx^2}+V(x)\psi(-x)=E\psi(-x) \tag{2}$$

Thus $\psi(-x)$ satisfies the same equation as $\psi(x)$ for an even potential, so another pair of solutions must be (since the equation is linear) $\psi(-x)\pm\psi(x)$. Taking the + sign, we get the function $\psi_+(x)=\psi(-x)+\psi(x)=\psi_+(-x)$, which is an even function. Taking the $-$ sign we get $\psi_-(x)=\psi(-x)-\psi(x)=-\psi_-(x)$, which is an odd function. Thus the general solution is a linear combination of even and odd functions.

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