EXTENDED UNCERTAINTY PRINCIPLE

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In deriving the uncertainty principle for two operators $\hat{A}$ and $\hat{B}$, we found that

$$\sigma_A^2\sigma_B^2 \ge |\langle f|g\rangle|^2 \tag{1}$$

where

$$|f\rangle = \left|(\hat{A}-\langle A\rangle)\Psi\right\rangle \tag{2}$$

$$|g\rangle = \left|(\hat{B}-\langle B\rangle)\Psi\right\rangle \tag{3}$$

From there, we obtained the final inequality by considering only the imaginary part of $z=\langle f|g\rangle$, and got the uncertainty relation

$$\sigma_A^2\sigma_B^2 \ge \left(\frac{1}{2i}\left\langle[\hat{A},\hat{B}]\right\rangle\right)^2 \tag{4}$$

If we retain the real part of z as well, we can get a stronger condition. We need to calculate $\Re(z)=(z+z^*)/2$, which we can do using results from the earlier post. We found there that

$$\langle f|g\rangle = \left\langle\hat{A}\hat{B}\right\rangle - \langle A\rangle\langle B\rangle \tag{5}$$

$$\langle g|f\rangle = \left\langle\hat{B}\hat{A}\right\rangle - \langle A\rangle\langle B\rangle \tag{6}$$

Therefore,

$$\frac{1}{2}(z+z^*) = \frac{1}{2}(\langle f|g\rangle+\langle g|f\rangle) \tag{7}$$

$$= \frac{1}{2}(\langle\hat{A}\hat{B}\rangle+\langle\hat{B}\hat{A}\rangle-2\langle A\rangle\langle B\rangle) \tag{8}$$

and the extended uncertainty principle is

$$\sigma_A^2\sigma_B^2 \ge \left(\frac{1}{2i}\left\langle[\hat{A},\hat{B}]\right\rangle\right)^2 + \frac{1}{4}\left(\langle\hat{A}\hat{B}\rangle+\langle\hat{B}\hat{A}\rangle-2\langle A\rangle\langle B\rangle\right)^2 \tag{9}$$

In the special case where $\hat{B} = \hat{A}$, then $[\hat{A}, \hat{B}] = 0$ and the second term reduces to $2\langle A^2\rangle - 2\langle A\rangle^2 = 2\sigma_A^2$, so the condition becomes $\sigma_A^4 \geq \sigma_A^4$, which doesn't tell us anything new.