HARMONIC OSCILLATOR - FIRST ORDER PERTURBATION

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This is a simple example of applying first order perturbation theory to the harmonic oscillator. The energy levels of an unperturbed oscillator are

$$E_{n0}=\left(n+\frac{1}{2}\right)\hbar\omega \tag{1}$$

where $\omega=\sqrt{k/m}$ and the potential is $V=\frac{1}{2}kx^2$. If we perturb the potential by changing k slightly, so the new potential is

$$V'=\frac{1}{2}\left(1+\epsilon\right)kx^2 \tag{2}$$

then, of course, it's easy to find the exact energy levels just by changing k in the original formula:

$$E_n=\left(n+\frac{1}{2}\right)\hbar\omega' \tag{3}$$

$$=\left(n+\frac{1}{2}\right)\hbar\sqrt{\frac{k\left(1+\epsilon\right)}{m}} \tag{4}$$

We can expand the square root in a power series:

$$E_n=\left(n+\frac{1}{2}\right)\hbar\sqrt{\frac{k}{m}}\left[1+\frac{\epsilon}{2}-\frac{\epsilon^2}{8}+\ldots\right] \tag{5}$$

From first order perturbation theory, the change to the energy is (since the perturbation in the potential here is $V'-V=\frac{1}{2}\epsilon kx^2=\epsilon V$):

$$E_{n1}=\left\langle n0\right|\epsilon V\left|n0\right\rangle \tag{6}$$

We could do the integral implied here, but we've already worked out the mean value of the potential for the harmonic oscillator using the virial theorem, and we know that $\langle V\rangle=\langle T\rangle=E_n/2$ so

$$E_{n1} = \frac{\epsilon}{2} E_{n0} \tag{7}$$

$$= \frac{\epsilon}{2}\left(n+\frac{1}{2}\right)\hbar\sqrt{\frac{k}{m}} \tag{8}$$

This is the first order term in ϵ in the series expansion above.

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