HYDROGEN ATOM - WAVE FUNCTION EXAMPLES

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A few more examples of working out the hydrogen atom wave functions. Using the formulas in the last example, we can get R_{20}. The recursion formula for $n=2$, $l=0$ is

$$c_{j+1}=\frac{2(j+1)-4}{(j+1)(j+2)}c_j \tag{1}$$

The series has 2 terms, and we get $c_1=-c_0$, so

$$R_{20}(r)=\frac{1}{r}u_{20}(r) \tag{2}$$

$$=\frac{1}{r}\rho e^{-\rho}v_{20}(\rho) \tag{3}$$

$$=\frac{1}{2a}e^{-r/2a}c_0\left(1-\frac{r}{2a}\right) \tag{4}$$

To find c_0 we normalize the radial function:

$$\int_0^\infty r^2|R_{20}(r)|^2dr=\int_0^\infty c_0^2r^2\left[\frac{1}{2a}\left(1-\frac{r}{2a}\right)\right]^2e^{-r/a}dr \tag{5}$$

$$=\frac{a}{2}c_0^2 \tag{6}$$

$$=1 \tag{7}$$

So $c_0=\sqrt{2/a}$ and $R_{20}(r)$ is

$$R_{20}(r)=\frac{1}{\sqrt{2}a^{3/2}}e^{-r/2a}\left(1-\frac{r}{2a}\right) \tag{8}$$

The complete wave function is then

$$\psi_{200}=R_{20}(r)Y_{00}(\phi,\theta) \tag{9}$$

$$=\frac{1}{\sqrt{8\pi}}a^{-3/2}\left(1-\frac{r}{2a}\right)e^{-r/2a} \tag{10}$$

For $R_{21}(r)$, we have

$$c_{j+1}=\frac{2(j+2)-4}{(j+1)(j+4)}c_j \tag{11}$$

This time, there is only a single term in the series, so we have

$$R_{21}(r) = \frac{1}{r}u_{21}(r) \tag{12}$$
$$= \frac{1}{r}\rho^2 e^{-\rho}v_{21}(\rho) \tag{13}$$
$$= \frac{r}{(2a)^2}e^{-r/2a}c_0 \tag{14}$$

Doing the normalization integral for $R_{21}(r)$ gives $c_0=\sqrt{2/3a}$ which gives the final result

$$R_{21}(r)=\frac{r}{2\sqrt{6}a^{5/2}}e^{-r/2a} \tag{15}$$

There are 3 wave functions corresponding to $n=2,\ l=1$, for which we need the spherical harmonics

$$Y_1^1 = -\left(\frac{3}{8\pi}\right)^{1/2}\sin\theta e^{i\phi} \tag{16}$$
$$Y_1^{-1} = \left(\frac{3}{8\pi}\right)^{1/2}\sin\theta e^{-i\phi} \tag{17}$$
$$Y_1^0 = \left(\frac{3}{4\pi}\right)^{1/2}\cos\theta \tag{18}$$

The three wave functions are thus

$$\psi_{211} = -\frac{1}{8\sqrt{\pi}}\frac{r}{a^{5/2}}e^{-r/2a}\sin\theta e^{i\phi} \tag{19}$$
$$\psi_{21-1} = \frac{1}{8\sqrt{\pi}}\frac{r}{a^{5/2}}e^{-r/2a}\sin\theta e^{-i\phi} \tag{20}$$
$$\psi_{210} = \frac{1}{4\sqrt{2\pi}}\frac{r}{a^{5/2}}e^{-r/2a}\cos\theta \tag{21}$$

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