

IMPULSE APPROXIMATION IN SCATTERING THEORY

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In classical scattering theory, the simplest approximation is the *impulse approximation*, in which a particle's path is assumed to be a straight line right through the scattering region, and the total impulse resulting from the component of force perpendicular to the particle's trajectory is calculated. This impulse I is assumed to be a small fraction of the particle's incoming horizontal momentum p , so the scattering angle should be small, and given approximately by

$$\theta \approx \arctan \frac{I}{p} \quad (1)$$

As an example, we'll apply the impulse approximation to Rutherford scattering of a charge q_1 travelling with kinetic energy E from another charge q_2 at rest. We assume q_1 has an *impact parameter* b (that is, if the particle didn't interact with the target, it would pass by with a closest approach distance of b).

The impulse is the change in momentum that a force produces over a given time, so the required impulse from the perpendicular component of a force is

$$I = \int F_{\perp} dt \quad (2)$$

We'll take q_1 's straight line trajectory to be along the x axis and place q_2 at $y = b$ on the y axis. Then the Coulomb force between q_1 and q_2 is

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \quad (3)$$

with a perpendicular component of

$$F_{\perp} = F \frac{b}{r} = \frac{q_1 q_2 b}{4\pi\epsilon_0 r^3} \quad (4)$$

The impulse is

$$I = \frac{q_1 q_2 b}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{dt}{r^3} \quad (5)$$

To convert this to an integral over r we can use the fact that q_1 's speed is constant at

$$v = \frac{dx}{dt} = \sqrt{\frac{2E}{m}} \quad (6)$$

so we have

$$x = \sqrt{r^2 - b^2} \quad (7)$$

$$dx = \frac{r dr}{\sqrt{r^2 - b^2}} = \sqrt{\frac{2E}{m}} dt \quad (8)$$

$$I = \frac{q_1 q_2 b}{4\pi\epsilon_0 \sqrt{2E/m}} \times 2 \int_b^\infty \frac{dr}{r^2 \sqrt{r^2 - b^2}} \quad (9)$$

where the r integral is over the range of r from its closest approach when $r = b$ out to infinity. We've doubled the integral to account for the incoming ($-\infty < x < 0$) and outgoing ($0 < x < \infty$) legs of the journey.

The integral evaluates to

$$\int_b^\infty \frac{dr}{r^2 \sqrt{r^2 - b^2}} = \left. \frac{\sqrt{r^2 - b^2}}{b^2 r} \right|_b^\infty \quad (10)$$

$$= \frac{1}{b^2} \quad (11)$$

$$I = \frac{q_1 q_2}{2\pi\epsilon_0 b \sqrt{2E/m}} \quad (12)$$

The incoming momentum is

$$p = m \frac{dx}{dt} = \sqrt{2mE} \quad (13)$$

so

$$\theta \approx \arctan \frac{I}{p} \quad (14)$$

$$= \arctan \left[\frac{q_1 q_2}{4\pi\epsilon_0 b E} \right] \quad (15)$$

$$b \approx \frac{q_1 q_2}{4\pi\epsilon_0 E} \cot \theta \quad (16)$$

$$\tan \theta \approx \frac{q_1 q_2}{4\pi\epsilon_0 b E} \quad (17)$$

The exact answer is

$$b = \frac{q_1 q_2}{8\pi\epsilon_0 E} \cot \frac{\theta}{2} \quad (18)$$

$$\tan \frac{\theta}{2} = \frac{q_1 q_2}{8\pi\epsilon_0 b E} \quad (19)$$

For small θ , $\tan \frac{\theta}{2} \approx \frac{\theta}{2}$ so

$$\frac{\theta}{2} \approx \frac{q_1 q_2}{8\pi\epsilon_0 b E} \quad (20)$$

$$\theta \approx \frac{q_1 q_2}{4\pi\epsilon_0 b E} \quad (21)$$

which is consistent with 17.