POSITION OPERATOR - EIGENFUNCTIONS

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We've seen that we can calculate the eigenfunctions of the momentum operator by solving a simple differential equation. Finding the eigenfunctions of the position operator requires a different approach since no differential equation is involved. Its eigenvalue equation is

$$\hat{x}\,|e_{x'}\rangle = x'\,|e_{x'}\rangle \tag{1}$$

where the $\hat{x}$ on the LHS is the position *operator*, even though it just multiplies its eigenfunction, and the x' on the RHS is a *constant* eigenvalue. Thus we're looking for an eigenfunction which has the property that when it's multiplied by x it gives back the same function multiplied by a constant x'.

At first glance, you might think that we can just set $x = x'$ and this equation is then true for any function. However, remember that we're trying to find a function such that multiplying it by the continuous variable x results in a *fixed* value x' multiplied by the same function. If this confuses you, think back to the hamiltonian and its eigenvalue equation $H\psi_n = E_n\psi_n$. Here we have a single operator H which, when operating on a particular function ψ_n gives back that same function multiplied by a constant E_n. If we give it a different eigenfunction, the *same* operator will multiply this eigenfunction by a *different* eigenvalue, with the eigenvalue depending on which eigenfunction is operated on by H.

In the case of the position operator, then, we want to find a function that, when operated on by the operator $\hat{x}$ gives back the same function multiplied by a particular value x'. This function can be zero everywhere except at $x = x'$, which leads us to using the delta function as the eigenfunction. This is because

$$x\delta(x-x') = x'\delta(x-x') \tag{2}$$

where, remember, the x on the LHS is a continuous variable and the x' on the RHS is a constant. The delta function picks out the one value of x where $x = x'$.

Thus we can say

$$|e_{x'}\rangle = A\delta(x-x') \tag{3}$$

for some constant A.

This function isn't square integrable, but if we try to calculate the inner product we get

$$\begin{aligned} \langle e_z | e_y \rangle &= |A|^2 \int \delta(x-z)\delta(x-y)dx & (4) \\ &= |A|^2 \delta(z-y) & (5) \end{aligned}$$

If we pick $A=1$, we get a sort of pseudo-orthonormality condition:

$$\langle e_z | e_y \rangle = \delta(z-y) \tag{6}$$

This is the same normalization as the one we used for the momentum operator.

REFERENCES

(1) Berman, Paul R. (2018), *Introductory Quantum Mechanics*, Springer, Chapter 5.
(2) Griffiths, David J. (2005), *Introduction to Quantum Mechanics*, 2nd Edition; Pearson Education, Chapter 3.

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