REAL SOLUTIONS TO THE SCHRÖDINGER EQUATION

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The time-independent Schrödinger equation can be solved by separation of variables, with the spatial part satisfying

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+V(x)\psi=E\psi \tag{1}$$

with $V(x)$ being the potential function and E being one of the allowable energies.

There will always be a ψ that is a real solution of this equation. This follows from the fact that if we have a function $\psi(x)$ that is complex and solves 1 then, since $V(x)$ and E are real, the complex conjugate $\psi^*(x)$ will also solve the equation. This follows from taking the complex conjugate of 1:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi^*}{dx^2}+V^*(x)\psi^*=E^*\psi^* \tag{2}$$

which can be written, using $V(x)$ and E being real, as

$$-\frac{\hbar^2}{2m}\frac{d^2\psi^*}{dx^2}+V(x)\psi^*=E\psi^* \tag{3}$$

Since the equation is linear, linear combinations of solutions are also solutions, so $\psi+\psi^*$ and $i(\psi-\psi^*)$ are also solutions, and both these are real functions. This, of course, doesn't mean that real functions are the only ones worth considering, but it does say that if there are any solutions at all, some of them must be real.

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