SCHRÖDINGER EQUATION - MINIMUM ENERGY

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We've seen that the time-independent Schrödinger equation can be separated into two ordinary differential equations, one in space and one in time. We've also seen that the solution of the spatial equation can be taken to be real. Starting with the spatial Schrödinger equation we can also derive a condition on the energy E.

$$\frac{d^2\psi}{dx^2} = \frac{2m}{\hbar^2}(V-E)\psi \tag{1}$$

If $E < V_{\text{min}}$, the minimum value of the potential, then $V - E > 0$ for all x. This means that ψ and ψ'' have the same sign everywhere. If ψ has a maximum, from elementary calculus we must have $\psi'' < 0$ so at the point of the maximum ψ itself must be negative. Similarly, any minima of ψ must occur where ψ is positive. Therefore, ψ cannot tend to 0 as $x \to \infty$ so it can't be normalized. Thus any physically acceptable solution must have $E > V_{min}$. This condition is the analog of the classical condition that the energy (kinetic plus potential) of a particle can't be less than the minimum of the potential. That is, a particle at rest at the bottom of a potential well has the lowest possible energy.

COMMENTS

Tianluo_Qi (Eastern). 24 Jun 2018 14:47.

All the arguments here are based on the hypothesis that the state function has a maximum or minimum. If it doesn't, or if the state function doesn't even have a secondary derivative at the extremum point at all, values of the state function and its secondary derivative can have the same sign, but it's normalizable as well. As an example of the latter kind, consider the state function of the bound state of a delta function well, namely, $\sqrt{a}e^{-a|x|}$.

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