

**SELECTION RULES IN SPONTANEOUS EMISSION -  
TRANSITION BETWEEN SPHERICALLY SYMMETRIC STATES  
NOT ALLOWED**

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When deriving the selection rules for states connected by spontaneous emission of a photon in systems with a spherically symmetric potential we found that the  $l$  quantum numbers of the starting and ending states had to be related by

$$\left[ (l' + l + 1)^2 - 1 \right] \left[ (l' - l)^2 - 1 \right] = 0 \quad (1)$$

One solution is  $l' = l \pm 1$  (which makes the second factor 0), but the solution  $l' = l = 0$  also seems acceptable, since it makes the first factor 0. However, in the latter case we must have  $m' = m = 0$  (since the quantum number  $m$  must satisfy  $-l \leq m \leq +l$ ) so all matrix elements in the transition rate formula must be of the form

$$\langle \psi_b | \mathbf{r} | \psi_a \rangle = \langle n'00 | \mathbf{r} | n00 \rangle \quad (2)$$

When  $l = m = 0$ , the angular part of the wave function is given by the spherical harmonic

$$Y_0^0 = \frac{1}{\sqrt{4\pi}} \quad (3)$$

so the angular integrals in 2 are integrals over  $x$ ,  $y$  and  $z$  expressed in spherical coordinates (the wave functions are both independent of the angular coordinates  $\theta$  and  $\phi$ ). We therefore have for the angular integrals of the  $x = r \sin \theta \cos \phi$  component

$$\int_0^\pi \int_0^{2\pi} \sin^2 \theta \cos \phi d\phi d\theta = 0 \quad (4)$$

For  $y = r \sin \theta \sin \phi$  we get

$$\int_0^\pi \int_0^{2\pi} \sin^2 \theta \sin \phi d\phi d\theta = 0 \quad (5)$$

and for  $z = r \cos \theta$  we get

$$\int_0^\pi \int_0^{2\pi} \sin \theta \cos \theta d\phi d\theta = 0 \quad (6)$$

Therefore the dipole moment matrix element is identically zero if  $l' = l = 0$ :

$$\mathbf{p} = q \langle \psi_b | \mathbf{r} | \psi_a \rangle = 0 \quad (7)$$

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