

SPHERICAL HARMONIC USING THE RAISING OPERATOR

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Earlier, we found Y_2^1 using associated Legendre functions:

$$Y_2^1(\theta, \phi) = \sqrt{\frac{15}{8\pi}} e^{i\phi} \sin\theta \cos\theta \quad (1)$$

We can use the raising operator to find Y_2^2 from this. In spherical coordinates, the raising operator is

$$L_+ = \hbar e^{i\phi} \left[\frac{\partial}{\partial\theta} + i \cot\theta \frac{\partial}{\partial\phi} \right] \quad (2)$$

Applying the raising operator to Y_2^1 gives

$$L_+ Y_2^1 = -\sqrt{\frac{15}{8\pi}} \hbar e^{i\phi} ((\cos^2\theta - \sin^2\theta)e^{i\phi} - \cot\theta \sin\theta \cos\theta e^{i\phi}) \quad (3)$$

$$= \sqrt{\frac{15}{8\pi}} \hbar e^{2i\phi} \sin^2\theta \quad (4)$$

$$= A_2^1 Y_2^2 \quad (5)$$

where A_2^1 is the constant derived earlier for the raising operator:

$$A_l^m = \hbar \sqrt{(l-m)(l+m+1)} \quad (6)$$

In this case

$$A_2^1 = 2\hbar \quad (7)$$

Therefore

$$Y_2^2 = \sqrt{\frac{15}{32\pi}} e^{2i\phi} \sin^2\theta \quad (8)$$