SPIN 1

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We can apply the methods for analyzing spin 1/2 to higher spin states as well. Here we'll have a look at spin 1. For spin 1, we know there will be 3 eigenstates of S_z ($m=1,0,-1$). Following the derivation for spin 1/2 we have

$$\chi = a\chi_1 + b\chi_0 + c\chi_{-1} = a\left(\begin{array}{c}1\\0\\0\end{array}\right) + b\left(\begin{array}{c}0\\1\\0\end{array}\right) + c\left(\begin{array}{c}0\\0\\1\end{array}\right) \tag{1}$$

First, we know that $S^2\chi_j = \hbar^2 s(s+1)\chi_j = 2\hbar^2\chi_j$ for $j=1,0,-1$, so

$$S^2 = 2\hbar^2\left(\begin{array}{ccc}1&0&0\\0&1&0\\0&0&1\end{array}\right) \tag{2}$$

Since $S_z\chi_j = j\hbar\chi_j$ for $j=1,0,-1$

$$S_z = \hbar\left(\begin{array}{ccc}1&0&0\\0&0&0\\0&0&-1\end{array}\right) \tag{3}$$

To get S_x and S_y we first get the raising and lowering matrices S_+ and S_-. The actions of these matrices on the eigenspinors is

$$S_\pm |s\, m_s\rangle = \hbar\sqrt{s(s+1) - m(m\pm 1)}\,|s\, m_s \pm 1\rangle \tag{4}$$

So we get $S_+\chi_1 = 0$, $S_+\chi_0 = \sqrt{2}\hbar\chi_1$ and $S_+\chi_{-1} = \sqrt{2}\hbar\chi_0$, from which we can write

$$S_+ = \sqrt{2}\hbar\left(\begin{array}{ccc}0&1&0\\0&0&1\\0&0&0\end{array}\right) \tag{5}$$

By a similar argument we get

$$S_- = \sqrt{2}\hbar \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \tag{6}$$

We can now get S_x and S_y

$$S_x = \frac{1}{2}(S_+ + S_-) = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{7}$$

$$S_y = \frac{1}{2i}(S_+ - S_-) = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \tag{8}$$

As a check, note that the eigenvalues of all of S_z, S_x and S_y are $\hbar$, 0 and $-\hbar$, as can be checked by direct calculation. For example, for S_x:

$$\begin{vmatrix} -\lambda & \frac{\hbar}{\sqrt{2}} & 0 \\ \frac{\hbar}{\sqrt{2}} & -\lambda & \frac{\hbar}{\sqrt{2}} \\ 0 & \frac{\hbar}{\sqrt{2}} & -\lambda \end{vmatrix} = -\lambda\left(\lambda^2 - \frac{\hbar^2}{2}\right) - \frac{\hbar}{\sqrt{2}}\left(-\lambda\frac{\hbar}{\sqrt{2}}\right) = 0 \tag{9}$$

This gives the cubic equation

$$-\lambda^3 + \lambda\hbar^2 = 0 \tag{10}$$

which has the roots $\lambda = 0, \pm\hbar$.