UNCERTAINTY PRINCIPLE FOR POSITION AND ENERGY

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In this post we'll have a look at an example involving the uncertainty principle. This makes use of the generic uncertainty principle for two observables A and B:

$$\sigma_A^2\sigma_B^2 \geq \left(\frac{1}{2i}\left\langle [\hat{A},\hat{B}]\right\rangle\right)^2 \tag{1}$$

We can calculate an expression for a 'position-energy' uncertainty relation. For this we'll need the commutator $\left[\hat{x},\hat{H}\right]$, so we work that out first. Since H involves a derivative, we use a test function g on which to operate:

$$\begin{aligned}
\left[\hat{x},\hat{H}\right] g &= -\frac{\hbar^2}{2m}x\frac{\partial^2 g}{\partial x^2}+xVg+-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}(xg)-xVg && (2)\\
&= \frac{\hbar^2}{2m}\left(-x\frac{\partial^2 g}{\partial x^2}+2\frac{\partial g}{\partial x}+x\frac{\partial^2 g}{\partial x^2}\right) && (3)\\
&= \frac{\hbar^2}{m}\frac{\partial g}{\partial x} && (4)\\
&= \frac{i\hbar}{m}(pg) && (5)
\end{aligned}$$

From 1 with $\hat{A}=\hat{x}$ and $\hat{B}=\hat{H}$ we have

$$\begin{aligned}
\sigma_x^2\sigma_H^2 &\geq \left(\frac{1}{2i}\langle[\hat{x},\hat{H}]\rangle\right)^2 && (6)\\
&= \frac{\hbar^2}{4m^2}\langle p\rangle^2 && (7)
\end{aligned}$$

so the uncertainty principle here becomes

$$\sigma_x\sigma_H \geq \frac{\hbar}{2m}|\langle p\rangle| \tag{8}$$

For a stationary state, this doesn't tell you much because the average position of the particle doesn't change, so $\langle p\rangle = 0$. For linear combinations of stationary states, though, the average momentum will not be zero, so this condition is more than trivial.

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