BLACK HOLE ENTROPY - IT'S PRETTY LARGE

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The entropy of a solar mass black hole is given by

$$S=\frac{4\pi k_{B}G}{\hbar}M_{\odot} \tag{1}$$

How does this compare with the entropy of objects at 'normal' density and room temperature? In the latter case, the entropy is usually around $S_{n}=Nk_{B}$ where N is the number of particles making up the object. If we consider a solar mass black hole, and suppose that it is made up exclusively of ionized hydrogen (protons + electrons), we can get an estimate of the energy. The number of particles in such an object is

$$N=2\frac{M_{\odot}}{M_{H}} \tag{2}$$

where M_{H} is the mass of a hydrogen atom, and the factor of 2 accounts for 2 particles (electron + proton) per atom.

The ratio S/Nk_{B} is then

$$\frac{S}{Nk_{B}}=\frac{4\pi GM_{\odot}M_{H}}{2\hbar} \tag{3}$$

In GR units, this is a dimensionless quantity, since $\hbar$ has dimensions of kg m and G has dimensions of m kg^{-1}. In GR units, the values are

$$\hbar = 3.5153\times10^{-43}\text{ kg m} \tag{4}$$

$$G = 7.426\times10^{-28}\text{ m kg}^{-1} \tag{5}$$

$$M_{\odot} = 1.989\times10^{30}\text{ kg} \tag{6}$$

$$M_{H} = 1.66\times10^{-27}\text{ kg} \tag{7}$$

So we get

$$\frac{S}{Nk_{B}}=4.38\times10^{19} \tag{8}$$

The entropy of a black hole is vastly more per particle than in 'normal' matter.