DOPPLER EFFECT AND FOUR-MOMENTUM

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The Doppler effect can be derived from the Lorentz transformation of momentum for a photon. The energy and wavelength of a photon are related by Planck's formula

$$E_0 = \frac{h}{\lambda_0} \tag{1}$$

where h is Planck's constant and λ_0 is the wavelength.

In the rest frame of the photon's source, we have

$$\mathbf{p}_0 = [E_0, E_0] = E_0 [1, 1] \tag{2}$$

If we now transform to a frame moving at speed v away from the source, we get

$$\mathbf{p}_v = E_0 \gamma [1 - v, 1 - v] \tag{3}$$

$$= E_0 \sqrt{\frac{1-v}{1+v}} [1, 1] \tag{4}$$

Thus the transformed energy is $E_v = E_0 \sqrt{\frac{1-v}{1+v}}$ and the new wavelength is

$$\lambda_v = \sqrt{\frac{1+v}{1-v}} \lambda_0 \tag{5}$$

which is the usual Doppler red-shift formula.

Another way of getting this is to consider the scalar product $-\mathbf{p} \cdot \mathbf{u}_{\text{obs}}$ where $\mathbf{u}_{\text{obs}}$ is the four-velocity of an observer and $\mathbf{p}$ is the four-momentum of a passing object. Since the scalar product is invariant under a Lorentz transformation, we can work it out in the rest frame of the observer, where $\mathbf{u}_{\text{obs}} = [1, 0, 0, 0]$. In this case, we get

$$-\mathbf{p} \cdot \mathbf{u}_{\text{obs}} = -[E, p_x, p_y, p_z] \cdot [1, 0, 0, 0] \tag{6}$$

$$= E \tag{7}$$

That is, the scalar product is the energy of the object as seen by the observer.

In the case of the photon above, if we work out this product in the rest frame of the source, then in that frame $\mathbf{u}_{\text{obs}} = [\gamma, \gamma v]$ and $\mathbf{p} = \mathbf{p}_0 = E_0 [1, 1]$ so

$$E = -\mathbf{p} \cdot \mathbf{u}_{\text{obs}} \tag{8}$$

$$= -E_0 (\gamma v - \gamma) \tag{9}$$

$$= E_0 \sqrt{\frac{1-v}{1+v}} \tag{10}$$

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