

EINSTEIN TENSOR OF ZERO IMPLIES A ZERO RICCI TENSOR

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We wish to prove that $G^{ij} = 0$ if and only if $R^{ij} = 0$. The Einstein tensor is defined as

$$G^{ij} \equiv R^{ij} - \frac{1}{2}g^{ij}R \quad (1)$$

Clearly if the Ricci tensor $R^{ij} = 0$ then $G^{ij} = 0$ (since the curvature scalar is the contraction of the Ricci tensor: $R = g_{ij}R^{ij}$). To prove the converse, suppose $G^{ij} = 0$. Then we can multiply both sides by g_{ij} to get

$$0 = g_{ij}R^{ij} - \frac{1}{2}g_{ij}g^{ij}R \quad (2)$$

$$= R - 2R \quad (3)$$

$$R = 0 \quad (4)$$

since $g_{ij}g^{ij} = 4$. With $G^{ij} = 0$ and $R = 0$, 1 tells us that $R^{ij} = 0$. QED.