EMBEDDING A 2-D CURVED SURFACE IN 3-D - THE COSH

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Post date: 9 Sep 2022.

A second example embedding a curved-space 2-d metric in 3-d flat space. This time, our 2-d metric is

$$ds^2 = \cosh^2\left(\frac{r}{R}\right)dr^2 + r^2 d\phi^2 \tag{1}$$

Here, the ϕ component is already in the required form, so we can equate this to the cylindrical metric directly:

$$dz^2 + r^2 d\phi^2 + dr^2 = \cosh^2\left(\frac{r}{R}\right)dr^2 + r^2 d\phi^2 \tag{2}$$

$$\frac{dz}{dr} = \pm\left[\cosh^2\left(\frac{r}{R}\right) - 1\right]^{1/2} \tag{3}$$

$$= \pm\sinh\left(\frac{r}{R}\right) \tag{4}$$

This integrates directly to give

$$z = \pm R\cosh\left(\frac{r}{R}\right) \tag{5}$$

The upper lobe of this surface is shown in Fig. 1. (There is a lower lobe which is a mirror image of the upper one).

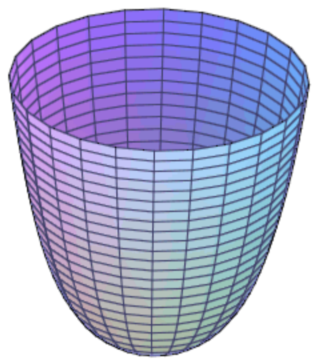

FIGURE 1. The surface $z = R\cosh\left(\frac{r}{R}\right)$.