

## EMBEDDING A 2-D CURVED SURFACE INTO 3-D - INVERSE COSH

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A final example embedding a curved-space2-d metric in 3-d flat space. This time, our 2-d metric is

$$ds^2 = d\rho^2 + (\rho^2 + b^2) d\phi^2 \quad (1)$$

where  $b$  is a constant.

To convert the  $\phi$  component to the required form of  $r^2 d\phi^2$ , we define  $r = \sqrt{\rho^2 + b^2}$ . Then

$$dr = \frac{\rho}{\sqrt{\rho^2 + b^2}} d\rho \quad (2)$$

$$= \frac{\sqrt{r^2 - b^2}}{r} d\rho \quad (3)$$

$$d\rho = \frac{r}{\sqrt{r^2 - b^2}} dr \quad (4)$$

The metric becomes:

$$ds^2 = \frac{r^2 dr^2}{r^2 - b^2} + r^2 d\phi^2 \quad (5)$$

We now equate this to the cylindrical metric:

$$dz^2 + r^2 d\phi^2 + dr^2 = \frac{r^2 dr^2}{r^2 - b^2} + r^2 d\phi^2 \quad (6)$$

$$\frac{dz}{dr} = \frac{b}{\sqrt{r^2 - b^2}} \quad (7)$$

This integral can be done with software or looked up, and is

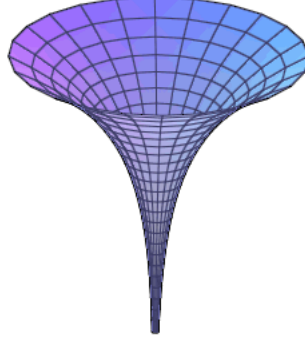


FIGURE 1. The surface  $z = b \operatorname{arcosh} \left( \frac{r}{b} \right)$ .

$$z = b \ln \left( r + \sqrt{r^2 - b^2} \right) \quad (8)$$

$$= b \ln \left[ b \left( \frac{r}{b} + \sqrt{\frac{r^2}{b^2} - 1} \right) \right] \quad (9)$$

$$= b \ln \left( \frac{r}{b} + \sqrt{\frac{r^2}{b^2} - 1} \right) + b \ln b \quad (10)$$

From tables of inverse hyperbolic functions, we see that this is equivalent to

$$z = b \operatorname{arcosh} \left( \frac{r}{b} \right) + b \ln b \quad (11)$$

We can ignore the last term as it is just a constant and serves only to raise or lower the surface as a whole. The surface (Fig. 1) is similar to Flamm's paraboloid.

#### PINGBACKS

Pingback: Wormhole metric