ESCAPE VELOCITY NEAR AN EVENT HORIZON

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Post date: 13 Jan 2023.

In Newtonian physics, the escape velocity of an object of mass m starting from a distance R from another mass M is defined as the velocity which the object must have if it is to arrive at infinity with a velocity of zero. In other words, the total energy E of the object (kinetic plus potential) must be zero at all times (since energy is conserved). The escape velocity v_e therefore satisfies

$$\frac{1}{2}mv_e^2 = \frac{GMm}{R} \tag{1}$$

$$v_e = \sqrt{\frac{2GM}{R}} \tag{2}$$

It turns out that the escape velocity calculated in the Schwarzschild metric obeys the same formula. An object at rest at infinity has a total energy per unit mass $e = 1$. We've also seen that the velocity of an object that moves radially as observed by an observer at R is

$$v_{\text{obs}}^2 = 1 - \frac{1}{e^2}\left(1 - \frac{2GM}{R}\right) \tag{3}$$

With $e = 1$, this gives the same formula for escape velocity

$$v_{\text{obs}} = v_e = \sqrt{\frac{2GM}{R}} \tag{4}$$

This formula applies only for $R > 2GM$, that is, outside the event horizon, and it predicts that as $R \to 2GM$, $v_e \to 1$, which is the basis for saying that a black hole will suck in anything that crosses the event horizon, so nothing with a non-zero rest mass can have a speed as great as 1.