FORCE IN RELATIVITY

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Force can be defined in relativity as the derivative of the spatial components of four-momentum with respect to ordinary (non-proper) time:

$$\mathbf{F}=\frac{d\mathbf{p}}{dt} \tag{1}$$

Superficially, this looks the same as Newton's second law, but in fact the formula for force is a bit more complex when written out in full. Using the definition of momentum, we have

$$\mathbf{F}=\frac{d}{dt}\left[\gamma m\mathbf{u}\right] \tag{2}$$

where

$$\gamma\equiv\frac{1}{\sqrt{1-u^{2}/c^{2}}} \tag{3}$$

We have

$$\frac{d\gamma}{dt} = \frac{1}{2c^{2}\left(1-u^{2}/c^{2}\right)^{3/2}}\frac{d\left(\mathbf{u}\cdot\mathbf{u}\right)}{dt} \tag{4}$$

$$= \frac{\gamma^{3}}{2c^{2}}\left(2\mathbf{u}\cdot\mathbf{a}\right) \tag{5}$$

$$= \frac{\mathbf{u}\cdot\mathbf{a}}{c^{2}\left(1-u^{2}/c^{2}\right)^{3/2}} \tag{6}$$

where $\mathbf{a}\equiv\dot{\mathbf{u}}$ is the acceleration.

Returning to 2, we have

$$\mathbf{F} = m\mathbf{u}\frac{d\gamma}{dt} + \gamma m\dot{\mathbf{u}} \tag{7}$$

$$= \frac{m(\mathbf{u}\cdot\mathbf{a})\mathbf{u}}{c^2\left(1-u^2/c^2\right)^{3/2}} + \frac{m\mathbf{a}}{\sqrt{1-u^2/c^2}} \tag{8}$$

$$= \frac{m}{\sqrt{1-u^2/c^2}}\left[\mathbf{a} + \frac{(\mathbf{u}\cdot\mathbf{a})\mathbf{u}}{c^2-u^2}\right] \tag{9}$$

This formula reduces to the familiar $\mathbf{F} = m\mathbf{a}$ in the limit of small $\mathbf{u}$. However, if we wish to retain a fixed acceleration as $u \to c$, the required force becomes infinite. Or looked at another way, if we want the force to remain finite as $u \to c$, the acceleration must drop to zero. In other words, it's impossible to accelerate an object with a non-zero rest mass to the speed of light.

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