FOUR-MOMENTUM CONSERVATION IN THE ELECTRON-ELECTRON COLLISION

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The invariance of the scalar product under Lorentz transformations can be useful in working out energies of particles in collisions. For example, suppose we have one electron colliding with another electron (where the second electron is at rest in the lab) and producing a new electron-positron pair (so after the collision there are 4 particles: 3 electrons and a positron). What is the minimum energy of the incoming electron (in the lab frame) to achieve this?

First, we can look at the problem in the centre of mass (COM) frame. There, the two electrons travel towards each other at the same speed v and after the collision all 4 particles are at rest. From conservation of momentum we must have

$$(\mathbf{p}_1+\mathbf{p}_2)^2 = -(4m)^2 \tag{1}$$

where $\mathbf{p}_i$ is the momentum of electron i before the collision, and m is the mass of an electron or positron.

Multiplying this out, we get

$$\begin{aligned} p_1^2+p_2^2+2\mathbf{p}_1\cdot\mathbf{p}_2 &= -m^2-m^2+2\mathbf{p}_1\cdot\mathbf{p}_2 &(2)\\ &= -16m^2 &(3)\\ \mathbf{p}_1\cdot\mathbf{p}_2 &= -7m^2 &(4)\end{aligned}$$

Since the scalar product on the LHS is invariant, it must also be true in the lab frame. There, $\mathbf{p}_1' = [m,0]$ since the first electron is at rest, and $\mathbf{p}_2' = [E,p_x]$. Therefore

$$\begin{aligned} \mathbf{p}_1'\cdot\mathbf{p}_2' &= -mE &(5)\\ &= \mathbf{p}_1\cdot\mathbf{p}_2 = -7m^2 &(6)\\ E &= 7m &(7)\end{aligned}$$

This gives us the minimum energy of the incoming electron in the lab frame.

To work out the velocity of the incoming electron, we note that

$$E = \gamma m \tag{8}$$

$$\gamma = 7 = \frac{1}{\sqrt{1-v^2}} \tag{9}$$

$$1 - v^2 = \frac{1}{49} \tag{10}$$

$$v = \frac{4\sqrt{3}}{7} \tag{11}$$