FOUR-MOMENTUM OF PHOTONS

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The four-momentum is defined for a particle with a rest mass m as

$$\mathbf{p}=\gamma m\left[1,v_x,v_y,v_z\right] \tag{1}$$

where $\gamma=1/\sqrt{1-v^2}$.

For particles such as photons that have no rest mass, this formula obviously doesn't work. However, the first component of $\mathbf{p}$ is taken as the particle's energy, so if we make that identification for photons, we get

$$\mathbf{p}=E\left[1,v_x,v_y,v_z\right] \tag{2}$$

For any massless particle, $v=\sqrt{v_x^2+v_y^2+v_z^2}=1$, so

$$\mathbf{p}\cdot\mathbf{p}=E^2\left(v_x^2+v_y^2+v_z^2-1\right)=0 \tag{3}$$

making $\mathbf{p}$ a null vector.

Example 1. Suppose we have a positive pion π^+ (rest mass 140 MeV) at rest that decays into an antimuon μ^+ (rest mass 106 MeV) and a neutrino. Although neutrinos do have some mass, it is very small so we'll just assume it's massless so its momentum is (assuming it travels along the $+x$ axis):

$$\mathbf{p}_\nu=E_v\left[1,1,0,0\right] \tag{4}$$

For the muon we have

$$\mathbf{p}_\mu=E_\mu\left[1,v,0,0\right] \tag{5}$$

where $E_\mu=106\gamma$ MeV.

From conservation of 4-momentum we get from the energy component

$$E_\nu+E_\mu=E_\pi \tag{6}$$

$$E_\nu+106\gamma=140 \tag{7}$$

From the second component we get

$$E_\nu + vE_\mu = 0 \tag{8}$$
$$E_\nu + 106v\gamma = 0 \tag{9}$$

Subtracting these two equations we get

$$106\gamma(1-v) = 140 \tag{10}$$
$$\frac{1-v}{\sqrt{1-v^2}} = 1.32 \tag{11}$$
$$1-2v+v^2 = 1.744-1.744v^2 \tag{12}$$

This quadratic has only one acceptable solution (there is also $v=1$ which is spurious since it makes γ infinite), which is $v=-0.271$. This gives $\gamma = 1.0389$, from which we get the energies:

$$E_\nu = 140-106\gamma \tag{13}$$
$$= 29.88\text{ MeV} \tag{14}$$
$$E_\mu = 110.12\text{ MeV} \tag{15}$$

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