

## FOUR-MOMENTUM OF PHOTONS

Link to: [physicspages home page](#).

To leave a comment or report an error, please use the auxiliary blog and include the title or URL of this post in your comment.

Post date: 20 June 2021.

The four-momentum is defined for a particle with a rest mass  $m$  as

$$\mathbf{p} = \gamma m [1, v_x, v_y, v_z] \quad (1)$$

where  $\gamma = 1/\sqrt{1-v^2}$ .

For particles such as photons that have no rest mass, this formula obviously doesn't work. However, the first component of  $\mathbf{p}$  is taken as the particle's energy, so if we make that identification for photons, we get

$$\mathbf{p} = E [1, v_x, v_y, v_z] \quad (2)$$

For any massless particle,  $v = \sqrt{v_x^2 + v_y^2 + v_z^2} = 1$ , so

$$\mathbf{p} \cdot \mathbf{p} = E^2 (v_x^2 + v_y^2 + v_z^2 - 1) = 0 \quad (3)$$

making  $\mathbf{p}$  a null vector.

**Example 1.** Suppose we have a positive pion  $\pi^+$  (rest mass 140 MeV) at rest that decays into an antimuon  $\mu^+$  (rest mass 106 MeV) and a neutrino. Although neutrinos do have some mass, it is very small so we'll just assume it's massless so its momentum is (assuming it travels along the  $+x$  axis):

$$\mathbf{p}_\nu = E_\nu [1, 1, 0, 0] \quad (4)$$

For the muon we have

$$\mathbf{p}_\mu = E_\mu [1, v, 0, 0] \quad (5)$$

where  $E_\mu = 106\gamma$  MeV.

From conservation of 4-momentum we get from the energy component

$$E_\nu + E_\mu = E_\pi \quad (6)$$

$$E_\nu + 106\gamma = 140 \quad (7)$$

From the second component we get

$$E_\nu + vE_\mu = 0 \quad (8)$$

$$E_\nu + 106v\gamma = 0 \quad (9)$$

Subtracting these two equations we get

$$106\gamma(1 - v) = 140 \quad (10)$$

$$\frac{1 - v}{\sqrt{1 - v^2}} = 1.32 \quad (11)$$

$$1 - 2v + v^2 = 1.744 - 1.744v^2 \quad (12)$$

This quadratic has only one acceptable solution (there is also  $v = 1$  which is spurious since it makes  $\gamma$  infinite), which is  $v = -0.271$ . This gives  $\gamma = 1.0389$ , from which we get the energies:

$$E_\nu = 140 - 106\gamma \quad (13)$$

$$= 29.88 \text{ MeV} \quad (14)$$

$$E_\mu = 110.12 \text{ MeV} \quad (15)$$

#### PINGBACKS

Pingback: Four-momentum conservation in electron-positron annihilation

Pingback: Doppler effect and four-momentum