GRAVITOMAGNETIC ACCELERATION IS PERPENDICULAR TO VELOCITY

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The geodesic equation for a particle in a weak field becomes

$$\frac{d^2x^m}{dt^2} = \eta^{ml}\left[\frac{1}{2}\partial_l h_{tt} + u^i\left(\partial_l h_{it} - \partial_i h_{lt}\right)\right] \tag{1}$$

The second term is the gravitomagnetic term, so named because it produces an acceleration that is proportional to the particle's velocity, in analogy to the magnetic Lorentz force. It turns out that, like the magnetic force, the gravitomagnetic acceleration is always perpendicular to the velocity. We can see this by taking the scalar product of the acceleration with the velocity. We get

$$\eta^{ml}u^i\left(\partial_l h_{it} - \partial_i h_{lt}\right)u_m = \eta^{ml}u^i\left(\partial_l h_{it} - \partial_i h_{lt}\right)\eta_{mp}u^p \tag{2}$$

$$= \eta^{ml}\eta_{mp}\left(\partial_l h_{it} - \partial_i h_{lt}\right)u^i u^p \tag{3}$$

$$= \delta_p^l\left(\partial_l h_{it} - \partial_i h_{lt}\right)u^i u^p \tag{4}$$

$$= \left(\partial_p h_{it} - \partial_i h_{pt}\right)u^i u^p \tag{5}$$

$$= \partial_p h_{it} u^i u^p - \partial_i h_{pt} u^i u^p \tag{6}$$

$$= \partial_p h_{it} u^i u^p - \partial_p h_{it} u^p u^i \tag{7}$$

$$= 0 \tag{8}$$

where in the penultimate line we swapped the i and p indices in the last term. Since the scalar product of the acceleration and velocity is zero, they are perpendicular.

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